



# **DEVIATIONS OF ELASTIC GONIOMETRIC MODULE'S CENTER OF ROTATION CAUSED BY LOCATION OF THE APPLIED FORCE**

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#### **Abstract**

*Goniometric elastic stages provide high accuracy angular positioning along one or more rotation axes. One of the main accuracy characteristics is the deviation of the center of rotation. There are many factors that could impact this characteristic like material, geometry and position of the elastic guides, load, position of the applied force for the movement, manufacturing process, etc. As main factor that could have effect on design and accuracy of the elastic module, the position of the applied force e.g. position of the linear actuator and it effect on deviation of the center of rotation is subject of this paper. Based on the finite element analysis, different positions have been studied. These results are important as there are many cases where the position of the applied force can't be on the optimal theoretical position or there are different deviations during production or assembly that could have impact on the accuracy/deviation of the center of rotation.*

**Keywords:** elastic guides, angular positioning, center of rotation

### **INTRODUCTION**

 The deviation of the center of rotation of a goniometric positioning system is a critical characteristic that defines its ability to precisely control and measure angular movements. Its accuracy is influenced by several factors and is typically specified in terms of angular precision, repeatability, resolution, stability, load capacity and its effect on the accuracy. The design of the goniometric elastic module is monolithic, based on the four-link compliant mechanism. The monolithic design eliminates any error caused by assembly of the elastic elements. The developed goniometric system is shown on Fig. 1. The working principle of the goniometric module is as follows: the angular rotation is carried out as the realized displacement from the micrometric screw 3 is transmitted to the coupled elastic module 2 by means of a rod 5 with conical tips at both ends. Spring 4 provides the contacting force. The elastic module is fixed to the base 6 by supports 7. The plate 1 is fixed to the elastic module, which tilts at a certain angle depending on the movement of the micrometric screw 3.



*Fig.1. Micro-positioning goniometric compliant elastic module*

This plate is used to provide a larger useful area, as well as to fasten elements on it, therefore, for greater convenience, threaded holes are provided, through which the positioned or measured objects are fastened.

The scope of this paper is the analysis of the behavior of the module in case the position of the applied force is different. This could help during development of such a module to select the best position in case there are different constrains that are not allowing to select the optimal theoretical position.

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#### **EXPOSITION**

#### **Simulation analysis scheme**

The analysis scheme is developed so that it can represent the real application of the goniometric elastic module. In this simulation study the finite element method (FEM) has been used. The test scheme is shown on Fig. 2. The two developed modules have monolithic design with 3 and 1 couples of elastic guides coupled in trapezoidal shape. The module is loaded on the surface where the micrometric screw is contacting the elastic module and the nonmovable part of the module is fixed. For the





*Fig.2. FEM analysis scheme of the goniometric elastic modules*

- *a) with 3 pairs of elastic guides (two cuts in the middle)*
- *b) with 1 pairs of elastic guides (without cuts in the middle)*

elastic module with cuts in the middle the fixed geometry is on the top surface marked with green arrows but for the elastic module without cuts the fixed geometry is the bottom plate again marked with green arrow. This case study represents almost exactly the real behavior of the module in the application. Used design for the FEM calculation is simplified to improve the time for calculation. The vertical forces shown on the scheme represents the loading of the upper base where a sample or part/system for analysis could be mounted.

## **Deviation of the center of rotation in case different positions of the applied force is used**

A key parameter for the goniometric module is the position of the rotation axis. It is important during angle measurement or positioning and the error should be minimal. This error has impact directly on the accuracy of the system. Position of the center of rotation is influenced by many factors - position of applied force point (e.g. position of the micrometric screw), dimensions and geometry of the elastic guides, load capacity. As this deviation of the center of rotation is a key accuracy indicator, a study to show its behavior during application is performed in case different positions of the applied force are set.



*Fig.3. Positions of the applied force*

Deviation of the center of rotation is captured along X and Y axis. The positions and the values of the applied force are changed according the same scheme used in the study for the transfer curve (positions 1 to 3, Fig. 3).

Obtained results where deviation of the center of rotation along X and Y axis in case different position of the applied force e.g. different position of the micrometric screw and different angles are set are shown on the graphics Fig. 5-10.



*Fig. 4. Scheme for calculation of the center of rotation using coordinates of two points*

Calculation of the instantaneous center of rotation is based on the scheme shown on Fig. 4. The coordinates  $X_k$  and  $Y_k$  of the instantaneous center  $C_k$  of rotation in a given plane are determined based on the measured coordinates of two points of the working platform lying in this plane perpendicular to the axis of rotation in two positions of the platform before and after its rotation by an angle  $\varphi$  - point A<sub>1</sub> (X<sub>A1</sub>, Y<sub>A1</sub>) and point  $B_1$  ( $X_{B1}$ ,  $Y_{B1}$ ) and respectively point  $A_2$  ( $X_{A2}$ ,  $Y_{A2}$ ) and point  $B_2$  ( $X_{B2}$ ,  $Y_{B2}$ ) -(Fig. 4).

In this case calculation of the coordinates  $X_k$  and  $Y_k$  of the instantaneous center  $C_k$  of rotation is based on the following formulas:

$$
x_k = \frac{(x_M - y_M \cdot \text{tg } \alpha) \cdot \text{tg } \beta - (x_N - y_N \cdot \text{tg } \beta) \cdot \text{tg } \alpha}{\text{tg } \beta - \text{tg } \alpha}; \quad (1)
$$

$$
y_k = \frac{(x_M - y_M \cdot tg\alpha) - (x_N - y_N \cdot tg\beta)}{tg\beta - tg\alpha},
$$
 (2)

where:

$$
x_M = \frac{x_{A_1} + x_{A_2}}{2}; \quad y_M = \frac{y_{A_1} + y_{A_2}}{2}; \tag{3}
$$

$$
x_N = \frac{x_{B_1} + x_{B_2}}{2}; \quad y_N = \frac{y_{B_1} + y_{B_2}}{2}; \tag{4}
$$

$$
tg \alpha = \frac{y_{A_2} - y_{A_1}}{x_{A_2} - x_{A_1}}; \quad tg \beta = \frac{y_{B_2} - y_{B_1}}{x_{B_2} - x_{B_1}}.
$$
 (5)

Based on these formulas the calculated position of the instantaneous center of rotation on the zero position or in case there is no load on the goniometric module is 16,98 mm measured from the top plane of the working platform. This parameter should be taken into account when the goniometric module is used in the application. During the performed study the instantaneous center of rotation is calculated based on the taken coordinates of two points from the working platform during deformation of the monolithic elastic module. These coordinates are taken using the simulation analysis scheme shown on Fig. 2. And calculated deviation of the center of rotation is captured. For the purpose of the study three positions are taken – upper, lower and one in the middle (position 1, 2 and 3, Fig. 3). On Fig. 5 are shown the results of the study if the applied force is placed on position 1 for the both elastic modules – with cuts in the middle and without cuts. Deviation of the



*Fig. 5. Deviation of the axis of rotation along X axis for the two designed modules for pos. 1 1. Elastic module with 2 cuts in the middle*

*2. Elastic module without cuts*

center of rotation of the elastic module without cuts in the middle along X-axis is 0,32 mm, but the results for the elastic module with cuts in the middle show deviation of center of rotation along same axis less than 0,1 μm which relatively small difference (0,2 mm) but it is around 4 times less deviation resulting in higher accuracy; therefore, the paired module can guarantee good preservation of the position of center of rotation along X-axis. The graph clearly shows the difference in the rotation axis position error of the two modules, unlike the effect on the transfer curve, where there was almost no difference between the two. The situation along the Y-axis is different, where the two modules have approximately identical characteristics (Fig. 6). The deviation of the center of rotation is 0,05 mm with less than 9 % difference between the two designed modules. The elastic module has slightly better performance along Y-axis, but when calculate the deviation along Y and X-axis combined, the elastic module with cuts in the middle show better performance.





*2. Elastic module without cuts*

The same study has been done, but the position of the applied force has been changed (position 2 Fig. 3), the results are reflected in graphs 7 and 8 for the deviation of the center of rotation along X and Y-axis respectively. This is done in order to understand how the two modules change their behavior in case different positions of the applied force are used and whether there are situations where one module improves its performance over the other. On the graph (Fig. 7), the advantage of the elastic module with cuts in the middle is clearly seen, the X-axis error becomes approximately ten times smaller than that of the elastic module without cuts that is reaching 0,4 mm. The elastic module with cuts preserve its deviation in the same area under 0,1 mm Therefore, the influence of the position is more pronounced in the elastic module without cuts and the error in it is larger. This position is in the lower end of the elastic module, so in this point the deviation of the center of rotation is expectable to be the largest one.





- *1. Elastic module with 2 cuts in the middle*
- *2. Elastic module without cuts*

The results on the Y-axis are a little different than these on X axis (Fig. 8). The deviation of the center of rotation is comparable for the both elastic modules. The results are almost the same as these for the previous position of the applied force (position 1) and the deviation is reaching values around  $0.05$  mm with less than 9  $\%$ difference between the two designs with

slightly better performance of the elastic modules without cuts. The only difference that is observed is that the graph of the elastic module with two cuts preserve better its linearity. In addition to this if the deviation is calculated along both axes combined, the module with two cuts has less deviation.



*Fig. 8. Deviation of the axis of rotation along Y axis for the two designed modules for pos. 2*

- *1. Elastic module with 2 cuts in the middle*
- *2. Elastic module without cuts*

Similarly, the research was done for position 3 (Fig. 3) as a selected point for the applied force. The results for the deviation of the center of rotation are reflected in graphs 9 and 10 for X and Y axis respectively.





*2. Elastic module without cuts*

At the top position (pos. 3), the deviation of the two elastic modules equalizes, the deviations of the module without cuts is decreasing, and the deviations of the module with cuts remain their values within the same, reaching 0,2 mm. The deviation of the center of rotation for the elastic module with cuts in the middle are comparable with the previous two studied cases and is reaching similar values around 0,1 μm. This means that the elastic module with cuts preserve its deviation along Xaxis better when it is used as a design for larger displacements.



*Fig. 10. Deviation of the axis of rotation along Y axis for the two designed modules for pos. 3 1. Elastic module with 2 cuts in the middle*

- *2. Elastic module without cuts*
- 

The deviation of the center of rotation along Y-axis for position 3 of the applied force is showing that there is almost no difference between the two designs. Both modules have deviations of approximately 0,045 mm. Moving the position of the applied force closer to the center of rotation is decreasing the working range of the modules e.g. obtained angle positioning and is minimizing the deviation of the center of rotation especially observed for the design without cuts.

#### **CONCLUSION**

Working range and deformations are larger if the position of the applied force, e.g. position of the micrometric screw is placed lower (large distance from the center of rotation) but the deviation of the position of the center of rotation is increasing. The elastic module with two cuts is less influenced in this regard and has less deviation in case the working range e.g. obtained angle positioning is enlarged.

The design with two cuts shows better performance in case larger angle displacement is needed with less deviation of the center of rotation.

The two elastic modules have linear transfer curve, but this with two cuts provides more accurate position of the center of rotation in various situations that may arise (most often inaccurate positioning of the micrometer screw from incorrect installation).

## **REFERENCE**

- [1] Ashby MF. Materials selection in mechanical design.2nd ed. Oxford: Butterworth-Heinemann; 1999.
- [2]Diakov D., Komarski D., Micro-positioning Module for Angular Orientation Position of the Axis of Rotation Analysis, (2021) 31st International Scientific Symposium Metrology and Metrology Assurance, MMA 2021, DOI: 10.1109/MMA52675.2021.9610873
- [3]Dichev, D., Koev, H., Bakalova, T., Louda, P. A Model of the Dynamic Error as a Measurement Result of Instruments Defining the Parameters of Moving Objects. Measurement Science Review, Volume 14, Issue 4, 2014, 183-189, ISSN 1335-8871, https://doi.org/10.2478/msr-2014-0025
- [4]Dichev, D., I. Zhelezarov, N. Madzharov. Dynamic Error and Methods for its Elimination in Systems for Measuring Parameters of Moving Objects. Transactions of Famena, vol. 45, issue 4, 2021, pp 55-70. DOI 10.21278/TOF.454029721
- [5]Georgiev, B., Karadzhov, T. Investigating the Repeatability of 3D Printers Using a Multi-Sensor Measurement System (2024) Vide. Tehnologija. Resursi - Environment, Technology, Resources, 3, pp. 65-69. DOI: 10.17770/etr2024vol3.8114
- [6]Dyakov D., I. Kalimanova, Opredelyane polozhenie na osta na rotatsiya na

goniometrichni pozitsioneri, Sbornik dokladi ot 1-va konferentsiya s mezhd. uchastie "Mashinoznanie i mashinni elementi", Sofiya, 4-6 Noemvri 2004

[7]Hui Tang, Yangmin Li and Jiming Huang, Design and analysis of a dual-mode driven parallel XY micromanipulator for micro/nanomanipulations, Proceedings of the Institution of Mechanical Engineers Part C Journal of Mechanical Engineering Science 1989-1996 (vols 203-210) 226(12):3043-3057,

DOI:10.1177/0954406212442272

- [8]ISO 230-7:2015 Test Code For Machine Tools. Geometric Accuracy Of Axes Of Rotation
- [9]Kim, Yong-Sik & Shi, Hongliang & Dagalakis, Nicholas & Gupta, Satyandra. (2016). Design of a MEMS-based motion stage based on a lever mechanism for generating large displacements and forces. Journal of Micromechanics and Microengineering. 26. 095008. 10.1088/0960-1317/26/9/095008
- [10] Michael F. Ashby Materials Selection In Mechanical Design 4Ed, Butterworth-Heinemann 2010, ISBN-13 978- 9380931722
- [11] Qian LU, Optimization Design of Flexible Micro-Displacement Amplification Mechanism Based on Parameters, School of Mechanical Engineering, Yancheng Institute of Technology, Yancheng City, Jiangsu Province 224051, P.R. China, https://doi.org/10.6180/jase.2015.18.4.05
- [12] Slocum A. Precision Machine Design. Englewood Cliffs, New Jersey: Prentice Hall; 1992. ISBN: 978-0872634923
- [13] Vasilev V., Nikolova H., Programno osiguryavane na sistemi za izmervane na otkloneniyata na formata i razpolozhenieto na povŭrkhninite i osite na detaĭlite, Sofiya, Softtreĭd, 2020, ISBN 978-954-334-231-0
- [14] Xu Pei1, Jingjun Yu, Guanghua Zong, Shusheng Bi, A Family of Butterfly Flexural Joints: Q-LITF Pivots, School of Mechanical Engineering and Automation, Beihang University, Beijing 100191, R.P. China, DOI:10.1115/DETC2011-48394
- [15] Zentner L, Linß S. Compliant Systems – Mechanics of Elastically Deformable Mechanisms, Actuators and Sensors. München: De Gruyter Oldenbourg; 2019, 166 p., ASIN: B06Y6FNSXB
- [16] Georgiev, B., Karadzhov, T., "Comparative Analysis оf Geometric Deviations in Contact Measuring Instruments for Control and Laser Contactless Scanning", In ENVIRONMENT. TECHNOLOGIES. RESOURCES. Proceedings of the International Scientific and Practical Conference, volume 3, 2023, June, pp. 306- 310. DOI: 10.17770/etr2023vol3.7182
- [16] D. Dichev et al., Mathematical Model for Increasing Accuracy when Measuring Linear Quantities in Conditions of External

Mechanical Impacts, 2023 XXXIII International Scientific Symposium Metrology and Metrology Assurance (MMA), Sozopol, Bulgaria, 2023, pp. 1-6, doi: 10.1109/MMA59144.2023.10317908.

[17] Dichev, Dimitar & Diakov, Dimitar & Dicheva, Ralitza. (2022). Method for increasing the accuracy of linear measurements based on a measurementcomputational approach. AIP Conference Proceedings. 2505. 120008. 10.1063/5.0101084.