MULTISIM-BASED MODEL FOR UNIMORPH PIEZOELECTRIC ENERGY HARVESTERS

Constantinos Tramantzas
Technological Educational Institute of Eastern Macedonia and Thrace (TEI EMT)

Abstract
Piezoelectric transduction has received great attention for vibration-to-electric energy conversion in the last years. Cantilevered beams with piezoelectric layers are lately more and more frequently used in the role of energy harvesters. The development of piezoelectric energy harvesting systems needs accurate models for system behavior predictions and evaluations. The modeling approaches in the literature are primarily mathematically-based and they include coupled single degree-of-freedom (SDOF) models, approximate distributed parameter models (based on the Rayleigh-Ritz method) as well as some distributed parameter approaches that consider only single vibration mode with the ignorance of the backward coupling in the mechanical domain. The current work proposes a Multisim-based model which is capable of studying the electromechanical behavior of unimorph piezoelectric energy harvesters. In the modeling procedure, Euler-Bernoulli beam theory is used as the procedure for obtaining the model parameters is described. The parametric simulation of a specific cantilevered piezoelectric energy harvester is done and the simulation results are analyzed. The comparison of the simulation results and those from analytical models, which are published in the literature shows that the proposed model is adequate. That means, the Multisim model successfully enables to simulate and analyze the electromechanical behavior of unimorph piezoelectric energy harvesters in their design process.

Keywords: energy harvesting; vibration mode; electromechanical behavior; Multisim model.

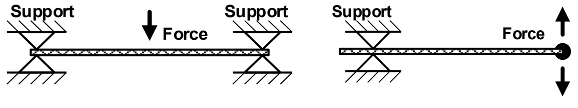
INTRODUCTION

The increasing ecological criteria and the dwindling quantity of energy resources on Earth dictate the necessity of developing new, efficient and renewable energy sources. One of the considered development paths is the creation of "self-powering" devices – kind of devices that are powered by naturally occurring phenomena that are unsuitable for large scale electric generation. The desirable trends are to replace or backup accumulators and rechargeable batteries with compact, effective and affordable electricity generating devices that are summed under the term Energy Harvesters (EH). The energy harvesters are operating on classical and known physical phenomena but the sources for the converted external energy are considered unconventional in the general sense.

EXPOSITION

A) Piezoelectric Energy Harvesters

There are various ways to collect energy from different sources in the environment which can vary from thermal conversion to using the random air turbulence flows [2]. Piezoelectric mediums are often considered when there is a possibility to harvest electricity from wasteful mechanical energy which in the most cases is present in moving systems as unused accompanying vibrations, motions or unbeneficial force projections. The reason for utilizing piezoelectric materials is their intrinsic high mechanical-to-electricity conversion ratio with which in theory substantial energy returns can be achieved. When considering the design of an energy harvester based on piezoelectric materials the initial step is to determine the operation mode and the second stage – to determine the placing of the active element.



a) b)

Fig. 1. *Mounting of the piezoelectric harvester*

There are two basic operational modes that are currently being put under development – namely the utilization of applied compression forces (Fig. 1, a) [3] or the usage of intrinsic vibrations that cannot be in some way diminished or damped (Fig. 1, b) [1].

The chosen operation mode determines the placing and mounting position of the harvester as the former mode is applied mainly in devices aimed at harvesting the human motions as the harvester materials are usually characterized with relatively high flexibility. The latter mode is usually applicable in different solutions in vibration rich environments which are likely to be present in the heavy industries but nothing prevents its usage in other applications as the active materials are usually defined with relatively high rigidity.

The vibrational harvesters are generally configured as some form of beam cantilever system, although some other resemble specialized spring structures. The beam configuration is used in connection to the fact that the rigidity of the active material in such configuration is not so important because of the passive substructure (layer) that can be optimized to meet the requirements for a specific application.

B) Basic Piezoelectric Beam Structure

A unimorph or monomorph harvester is a cantilever system that consists of one active (piezoelectric) layer and one inactive layer. The beam usually is formed from a layer of thin piezoelectric ceramic [2] (which provides the required stiffness to the resulting structure and in most cases some kind from PZT piezoelectric ceramics is used) and one or two layers of flexible, nonconductive material (usually some kind of plastic).

In Fig. 2, a variant of a unimorph harvester is shown. The harvester beam is assumed to be excited by the motion of its base, which is represented by translation in the transverse direction and small rotation. The harvester can be represented as a simply an Euler-Bernoulli uniform composite beam consisting of a PZT layer bonded to the substructure layer.

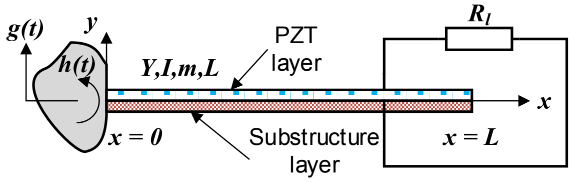


***Fig. 2.** Unimorph piezoelectric energy harvester*

Modeling the configuration as a uniform composite beam based on the theory of Euler-Bernoulli is a reasonable assumption since typical cantilevered piezoelectric energy harvesters are designed and manufactured as fairly thin beams and most of the commercially available unimorphs are thin structures. Deformations are assumed to be relatively small and the composite structure to exhibit linear-elastic material behavior.

The harvester beam is typically excited due to the motion of its base. If the translation and the small rotation of the base are denoted by *g(t)* and *h(t)*, respectively, as shown in Fig. 2, then the base motion $w_b(x,t)$ on the beam can be represented as follows:

$$w_b(x,t) = g(t) + h(t). \qquad (1)$$

After the appropriate transformations and utilizing the required assumptions, the electromechanically coupled ordinary differential equation for the modal response of the beam can be obtained as following:

$$\frac{d^2\eta_r(t)}{dt^2} + 2\zeta_r\omega_r\frac{d\eta_r(t)}{dt} + \omega_r^2\eta_r(t) + X_r v(t) = N_r(t)\text{'} \qquad (2)$$

where:

$\eta_r(t)$ – mechanical modal response;

ω_r – undamped natural frequency of the $r^{\text{-th}}$ mode;

ζ_r – mechanical damping ratio,

k_r – modal coupling term from the mechanical to electrical part;

X_r – modal coupling term from the electrical to mechanical part,

$v(t)$ – voltage across the resistive load;

$N_r(t)$ – modal mechanical forcing function.

It is possible to represent the electrical domain of the coupled system by very simple circuit as shown on Fig. 3.

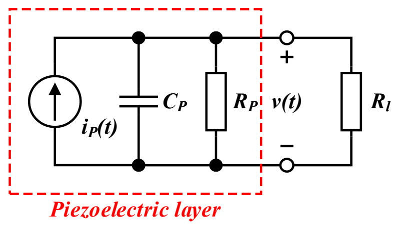


***Fig. 3.** Equivalent electrical circuit of the piezoelectric layer*

The parameter that are used to describe the equivalent circuit, are the following: $i_P(t)$ (current, generated in the piezoelectric layer), C_P (internal capacitance of the piezoelectric layer), R_P (internal resistance of the piezoelectric layer or else known as piezoelectric leakage resistance) and R_l (load resistance).

The simple circuit shown in Fig. 3 is the complete circuit of the electrical domain for a single resistive load case. This representation considers the electrical domain only and the electromechanical representation of the coupled system is actually a transformer because of the voltage feedback sent to the mechanical domain due to the piezoelectric coupling. Applying the Kirchhoff laws to the simplified equivalent electrical circuit, leads to the following:

$$C_p \frac{dv(t)}{dt} + \frac{v(t)}{R_l} + \frac{v(t)}{R_P} = i_P(t). \quad (3)$$

The internal capacitance, the internal resistance and the current source can be represented as:

$$C_p = \frac{\varepsilon_{33}^S bL}{h_P}, \quad (4)$$

$$R_P = \rho_P \frac{h_P}{bL}, \quad (5)$$

$$i_p(t) = \sum_{r=1}^{\infty} k_r \frac{d\eta_r(t)}{dt}, \quad (6)$$

where: ρ_P is the resistivity of the piezoceramic.

It's important to note, that the solutions of the represented equations vary depending on the way of the harvester excitation. For excitation the harvester, one of the following cases may be chosen:

- General Transient Base Motion (GTBM) – the excitation of the harvester is assumed to be due to its base motion in the form of translation in the transverse direction with small rotation, and it is not restricted to be harmonic in time.
- Harmonic Base Motion (HBM) – it is assumed the translation and small rotation of the beam to be harmonic in time, i.e., $g(t) = Y_0 e^{j\omega t}$ and $h(t) = \theta_0 e^{j\omega t}$, where Y_0 and θ_0 are the amplitudes of the base translation and rotation, respectively, and ω is the driving frequency.
- Harmonic Base Translation (HBT) – in most of the theoretical and experimental studies on piezoelectric energy harvesting, the base excitation is considered to be harmonic translation in the transverse direction and the beam is assumed to be not rotating, that is $h(t)$=0. This case of the beam excitation can be implemented in two ways – harmonic base translation at an arbitrary frequency and harmonic base translation around the natural frequency of the $r^{\text{-th}}$ mode.

C) Modeling Procedure

For behavior modeling of a unimorph piezoelectric energy harvester, a requirement of establishing the sequence of operations, that correspond to the formation and solving of the analytical model is needed, to be made possible to develop a procedure for a schematic modeling.

The proposed modeling procedure concerns the case of HBT in which the motion of the base is reduced to transverse displacement only (the base of the cantilever is assumed to be not rotating and is shown in *Table 1*.

Table 1: *Steps of the modeling procedure*

Step 1:	Assumption that the displacement of the base $w_b(t)$ (where $w_b(t) = g(t)$) is the input;
Step 2:	Differentiating the displacement (input) according to time for obtaining the velocity of the base;
Step 3:	Differentiating the velocity with accordance of time for obtaining the acceleration of the base;
Step 4:	Converting the acceleration of the base into the internal excitation term of the modal mechanical forcing function;
Step 5:	Converting the velocity of the base into the damping excitation of the modal mechanical forcing function;
Step 6:	Determining the modal coupling term X_r, which describes the connection between the electrical and mechanical systems;
Step 7:	Forming the right-hand side of the modal equation of motion by summing the components of mechanical excitation and subtracting from the resulting sum the force contribution from the applied/induced voltage in the piezoelectric layer;
Step 8:	Solving the modal equation of motion for the modal mechanical response of the beam;
Step 9:	Determining the first derivative of the modal mechanical response of the beam;
Step 10:	Representing the electrical domain of the coupled system by an equivalent circuit which consists of the model of piezoelectric layer connected in parallel to the load resistance;
Step 11:	Determining the relative harvester tip displacement for a given value of r;

D) Model for a Unimorph Piezoelectric Harvester

For the purpose of helping the design process and the behavior simulations of electrical circuits which contain piezoelectric energy harvesters, there is a need of creating SPICE compatible model.

The proposed modeling procedure (in *Table 1*) has been implemented in the form of a structural behavioral model in the Multisim environment (fully SPICE compliant). The developed schematic model for a unimorph piezoelectric energy harvester is shown in Fig. 4. The model includes both structural (electronic) elements (for example, R and C elements) and functional blocks (such as, differentiators, gyrators, etc). The electronic elements model a structure of a certain circuit components (for example, the equivalent circuit of the piezoelectric layer), while the functional blocks describe a certain type of device behavior.

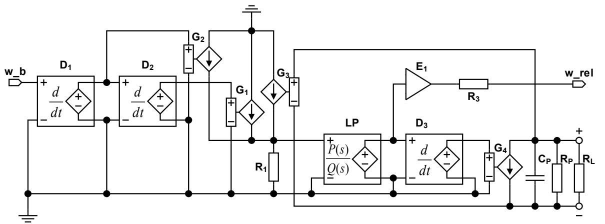


Fig. 4. *Multisim model of a piezoelectric unimorph cantilever harvester*

In the model, the displacement of the base w_b ($w_b(t)$,$w_b(t) = g(t)$) is taken as an input, where geometrical dimension is substituted by voltage.

The differentiator block $\mathbf{D_1}$ is used to get the first derivative of $w_b(t)$ (velocity $j\omega Y_0 e^{j\omega t}$) as the $\mathbf{D_2}$ is used to get the second – the acceleration ($-\omega^2 Y_0 e^{j\omega t}$). The used four gyrators ($\mathbf{G_{1\div4}}$) are represent as voltage controlled current source (VCCS). Gyrators $\mathbf{G_1}$ and $\mathbf{G_2}$ are used for conversion of the base acceleration and velocity to components of mechanical excitation (respectively N_r^m and N_r^c). Gyrator $\mathbf{G_3}$ models the force contribution from the applied/induced voltage in the piezoelectric layer. The Laplace block – **LP** is used to describe the mechanical system as a linear system and as it gives the modal amplitude $\eta(t)$. The differentiator $\mathbf{D_3}$ is used for obtaining the first derivative of the modal mechanical function. The piezoelectric layer can be modeled as a current source $\mathbf{G_4}$, internal capacitance $\mathbf{C_P}$ and internal resistance $\mathbf{R_P}$. The proportional relation between the relative vibratory motion of the beam and the modal response of the beam is modeled by gain block, labeled as $\mathbf{E_1}$. The block $\mathbf{E_1}$ can be considered as a two-pin voltage controlled voltage source (VCVS) and is used to convert the modal amplitude to a relative vibratory motion of the beam system.

In the model current stage of development, only one mode (i.e., r-th mode) can be simulated at a given time. To evaluate more modes simultaneously either the model needs to be expanded, or the results obtained separately for the different modes should be further processed together. Prior to the model usage, several parameters should be calculated as set parameters for the model blocks/elements. These parameters are design specific and they are considered to be constant during the simulation and correspond to the r-th mode.

The necessary parameter settings for the blocks/elements of the model are summarized in *Table 2*.

Table 2: *Parameter settings for the model elements*

Block/element			
Name	Label	Parameter	Value/Expression
Differentiator	D_1	Gain	1
	D_2	Gain	1
	D_3	Gain	1
Gyrators	G_1	Gain	$-m\int_{x=0}^{L}\phi_r(x)\,dx$
	G_2	Gain	$-c_a\int_{x=0}^{L}\phi_r(x)\,dx$
	G_3	Gain	$-k_r$
	G_4	Gain	$-X_r$
Laplace function	LP	s-variable Laplace expression/transfer function	$\dfrac{1}{(s^2+2\zeta_r\omega_r s+\omega_r^2)}$
Gain block	E_1	Gain	$\phi_r(L)$
Resistance	R_1	Resistance	1Ω
Internal resistance	R_P	Resistance	$\rho_P\dfrac{h_P}{bL}$
Internal capacitance	C_P	Capacitance	$\dfrac{\varepsilon_{33}^S bL}{h_P}$

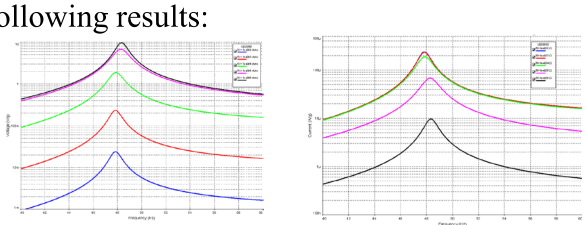
E) Simulation Research

Initially the simulations are run for determining the resonance frequencies of the harvester structure. The results for the first three vibration modes in the case of short circuit and open circuit resonance frequencies are shown in the *Table 3*.

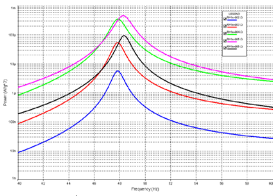
Table 3: *Resonance frequencies*

Frequency (Hz)	*Mode 1*	*Mode 2*	*Mode 3*
f_r^{sc} – short circuit	47.8	299.6	838.8
f_r^{oc} – open circuit	48.4	300.6	839.1
*Subscript **r** stands for the mode number*			

The simulations for *Mode 1* give the following results:



a) Voltage results b) Current results



c) Power output

Fig. 5. *Simulation results for current, voltage and power output for five different loads*

The comparison of the simulation results and those from similar analytical models [4, 5], which are published in the literature shows that the proposed model is adequate for simulation of piezoelectric energy harvesters in SPICE environment.

CONCLUSION

The current work propose SPICE compliant model that describes piezoelectric energy harvesters of cantilever beam type. The proposed model can be used in studying, developing and designing circuits in which this type of harvesters are included.

The model is based on the concept of Euler-Bernoulli uniform composite beam systems and electromechanical analogy in piezoelectric active mediums. The proposed model includes both structural and functional behavioral blocks and is composed in Multisim environment in which it had been tested.

The obtained results in comparison with the similar analytical models show that the developed Multisim model successfully enables to simulate and analyze the electromechanical behavior of unimorph piezoelectric energy harvesters in their design process and circuitry development.

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