

STATE ESTIMATION OF A SUMMER TOBOGGAN USING THE EXTENDED KALMAN FILTER

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Abstract

In this paper an approach for estimating the state of a rail-bound summer toboggan is presented. Usually those vehicles are accelerated by downhill force and controlled manually by moving a brake lever. To avoid accidents, an automatic brake system for controlling speed and distance was developed. In order to design a controller based on a given sensor configuration, a state estimation making use of not only sensor data but also a model of the plant could be useful. To accomplish that, a simplified physical model of a sled, including sensors and the brake actuator, was derived. Based on the model and a given sensor configuration, an observer in the form of an extended Kalman filter was designed. Both, the model and the observer, were implemented and tested using Simulink software. The simulation results show that the extended Kalman filter could be a useful approach for estimating the state of the system and furthermore be a suitable base for running the controller on a time critical embedded system.

Keywords: state estimation, observer design, extended Kalman filter

INTRODUCTION

Summer toboggan rides are one type of amusement rides, where one or two riders sit on a sled, which is accelerated by downhill force. A lever can be used to brake the sled by moving it towards the body, while when the lever is pushed forward, the brake is disabled and the sled accelerates. In the past, these toboggan rides worked without any electronics and the user was responsible to brake and avoid accidents. Because of regulations the maximum speed of these vehicles should be restricted. To accomplish that, mechanical systems were integrated in the sleds. These systems work well for a fixed maximum speed, but have large tolerances and are maintenance-consuming.

In order to avoid collisions, a system was required, which can detect and control the distance and a variable maximum speed. To achieve this goal, an automated system consisting of sensors, an embedded control unit and an automatic brake actuator was developed. First results have shown, that these components work well. By implementing a simple controller, the system is able to control the speed and can avoid accidents.

To improve the comfort for the riders, a more sophisticated control algorithm has to be

developed. An improvement of the control algorithm may be achieved by including an observer, which is able to reconstruct the state of the plant at each time step and so form a basis for designing a smooth controller, which is optimally not perceptible for the riders.

In this work, the extended Kalman filter (EKF) is applied on the system with a given sensor configuration for estimating the state of one sled. The distance between two sleds is supposed to be available to the controller by wireless communication. The controller design is not part of this work.

EXPOSITION

In the next part, a simplified physical model of the plant including the brake actuator of the sled is derived. In the subsequent part, the EKF is adopted to the problem. After that, the implementation of the model in Simulink is introduced followed by showing some simulation results. A discussion and conclusion including a view to further work mark the end of this report.

MODEL

A sled of the toboggan ride is accelerated by a downhill force F_D which depends on the

track slope α and the mass m of the sled including the riders. This force is calculated by

$$F_D = m g \sin \alpha, \quad (1)$$

where g is the acceleration of gravity. If there is no brake active, two relevant friction forces can be identified. First is the rolling resistance

$$F_R = c_R m g \cos \alpha, \quad (2)$$

with the coefficient of rolling friction c_R . The second one is the drag force

$$F_A = \frac{1}{2} \rho c_w A v^2, \quad (3)$$

where ρ is the density of the fluid, c_w the drag coefficient, A the cross sectional area and v is the speed of the sled.

The automatic brake system consists of a DC-motor which is connected to a permanent magnet. When the magnet is driven downwards over a conductive material, which is situated between the rails, eddy currents are generated and the sled is decelerated. The brake force can be approximated by the equation [1]

$$F_B = k_b b^2 v. \quad (4)$$

In this equation the constant k_b summarizes the electrical and mechanical parameters of the eddy current brake.

The resulting force F_{res} is given by the sum of forces:

$$F_{res} = F_D - F_R - F_A - F_B \quad (5)$$

The resulting acceleration can now be determined by inserting formulas 1-4. By replacing the term $\frac{1}{2} \rho c_w A$ with the parameter k_l , this leads to

$$a_{res} = g \sin \alpha - c_R g \cos \alpha - m^{-1} k_l v^2 - m^{-1} k_b b^2 v. \quad (6)$$

Because both, the manual brake and the motor, act on the brake magnet, the maximum is taken to model its position. Rewriting this system in state space form

$$d\mathbf{x}/dt = \mathbf{f}(\mathbf{x}, \mathbf{u}) \quad (7)$$

leads to the state equations

$$\begin{aligned} dx_1/dt &= x_2 \\ dx_2/dt &= g \sin x_3 - c_R g \cos x_3 - m^{-1} k_l x_2^2 \\ &\quad - m^{-1} k_b \max^2(x_4, u_2) x_2 \\ dx_3/dt &= d\alpha(x_1)/dt \\ dx_4/dt &= u_1 \end{aligned} \quad (8)$$

where x_1 is the position s of the sled and x_2 is its velocity. The slope α and the magnet depth are modeled as x_3 and x_4 respectively. The model inputs are the brake movement speed u_1 and the manually forced position of the brake magnet u_2 .

EXTENDED KALMAN FILTER

The realized system contains a position sensor on the bottom of the sled, which is able to detect binary position marks, which are placed all over the track in fixed distances. The velocity is not directly measurable. Because the track profile stays constant all the time, information about the slope is also available. The depth of the magnet is only measurable, when the lever is not manipulated, because it is loosely coupled to the motor. When a rider brakes manually, the magnet could be in a lower position than the motor position feedback suggests.

So the state vector is not always available only by using measurements. To reconstruct the state vector or even the speed of the sled, which is the primary control variable, an observer has to be implemented. In this case the extended Kalman filter is used for this purpose [2]. In contrast to other kinds of observers, i.e. the Luenberger observer [3], the EKF can use the unlinearized system without further approximations for its prediction step. Also the uncertainties can be modelled by adding process noise w or measurement noise v to the discrete state equations:

$$\begin{aligned} \mathbf{x}_{k+1} &= \mathbf{f}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{h}(\mathbf{x}_k, \mathbf{u}_k) + \mathbf{v}_k. \end{aligned}$$

For state estimation, the EKF uses a prediction and an update step. In the prediction step the next state of the system is estimated by using the current estimation and calculating the next state using the nonlinear but time discretized model.

IMPLEMENTATION

The plant and the EKF were both modeled using Simulink software. For implementing the EKF the Control System Toolbox was used [4]. This toolbox contains a block (Fig. 1), which implements a version of the EKF, where measurement inputs can be enabled or disabled to model sensors inputs, which are not available all the time. For the modelled system, this is very useful because the position measurements are not available in a continuous time interval. Also the measurement of the brake magnet is only assumed to be reliable when the brake lever is not manipulated. The manipulation of the lever can be detected by a sensor in binary form and this signal is used to disable the measurement of the motor feedback.

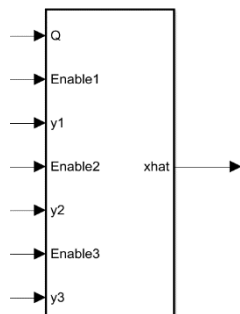


Fig. 1. The EKF block of the Simulink Control System Toolbox consists of an input for the covariance Q , measurement inputs y and Enable inputs. The output $xhat$ represents the state estimation.

For simulating the plant, the state equations (8) were modeled using standard Simulink blocks. Because in the real system the brake magnet position is restricted due to mechanical boundaries, a limited integrator was used. As control input a signal with a defined motor velocity was generated.

To emulate the position sensor signal, the first component of the continuous state vector from the model is used. In discrete position distances, a short high level signal is generated, which is detected using a rising edge detection block. This can be viewed as an emulation of an interrupt triggered capture.

The two remaining measurements (the slope angle and the brake position) are taken directly from the state vector, because they are assumed to be directly detectable. To each emulated sensor signal, measurement noise is

added. Furthermore, all sensor signals are fed through a rate transition block (zero order hold), which generates time discrete signals from continuous data.

For each sensor signal, an enable line for activating measurements in the EKF-block is generated. The position measurement is only available when a rising edge is detected. Also the slope angle is assumed to be available only on those track points. A manual brake action disables the measurement of the brake position.

Figure 2 shows the model including the EKF.

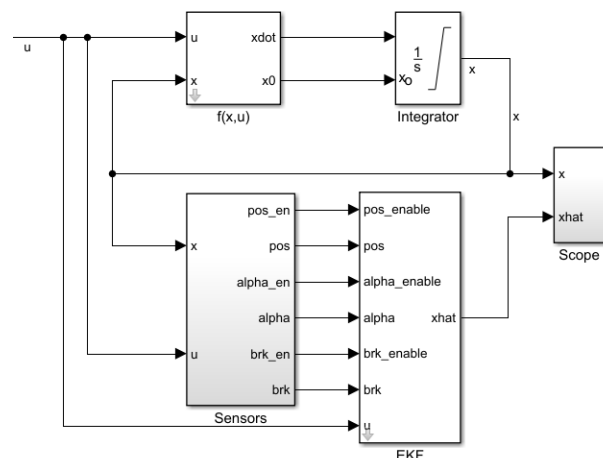


Fig. 2. The enhanced model with added sensor emulation and the EKF

SIMULATIONS AND RESULTS

For checking the plausibility of the model, the plant was simulated using three different track profiles, which can be analytically modelled: an inclined plane with constant slope, a varying slope using a cosine function and an exponentially falling function. The slopes were set regarding real track profiles. For checking the brake force, the parameter k_b was adopted to real data from test bench trials. Friction parameters were set to plausible values regarding comparable systems and materials. Figure 3 shows the simulation results for a constant slope with parameters set to $m = 200\text{kg}$, $c_R = 0.01$, $k_l = 0.36$ and $k_b = 10000$.

To prove the state estimation using the EKF, the simulation was carried out using different parameters in the simulated plant and the discretized model of the observer. Figure 4 shows the results for using the EKF. The plant parameters are equal to figure 3, but the EKF uses a parameter set of $m = 150\text{kg}$, $c_R = 0.02$,

$k_l = 0.54$ and $k_b = 8000$. The process covariance was set to $\text{diag}(0.01, 0.01, 1e-4, 1e-6)$ and the measurement covariance to $\text{diag}(0.01, 0.0001, 0.0001)$.

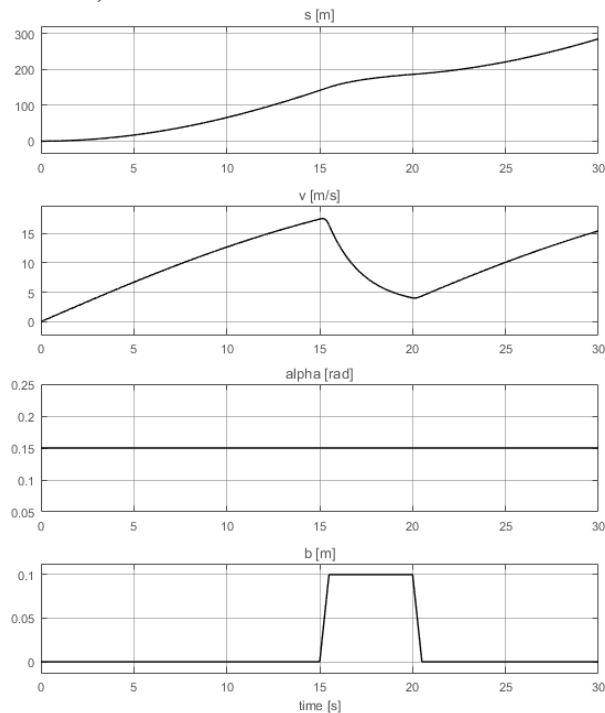


Fig. 3. Simulation of the plant model: A sled is driving down a track with constant slope. After 15 seconds the brake magnet is driven to maximum depth. After 20 seconds the brake is released. The graph shows the components of the state vector.

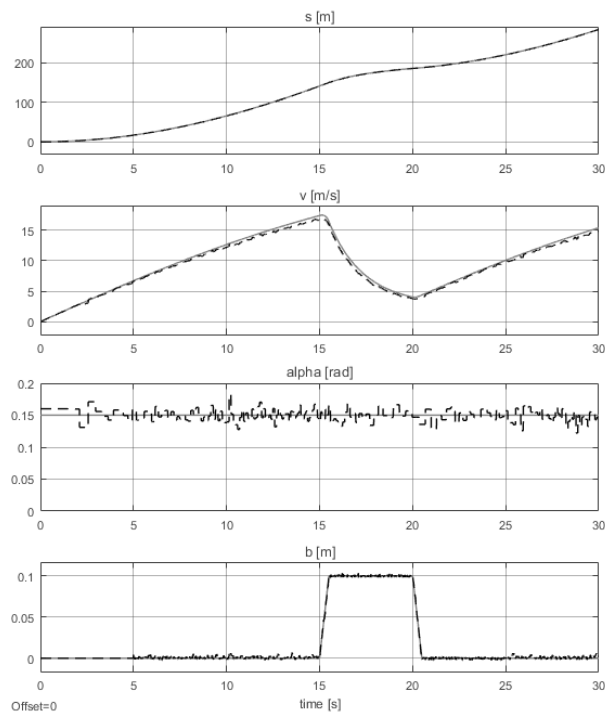


Fig. 4. Simulation of the plant (grey solid line) and the estimated state (black dashed line) using the EKF.

DISCUSSION AND CONSLUSIONS

Simulation results show, that the EKF seems to be an appropriate method to reconstruct the state vector from measurements with a given sensor configuration. Despite of using model generated data, the variation of parameters in a realistic range shows, that this approach could also work for real data. The plant was adopted to real data from brake measurements and friction parameters were used regarding similar systems and materials. The covariance matrix was estimated regarding expected variations of the sensor and plant data.

FURTHER WORK

The next step is to validate the model using real data recorded on a real track. To accomplish that, an embedded system with the possibility of storing raw data is being developed. After that, a controller for this nonlinear system has to be designed and tested.

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