

OPTIMIZATION OF THE 3P KEYS KERNEL PARAMETERS FOR INTERPOLACION OF AUDIO SIGNALS

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Abstract

The first part of this paper describes the 3P Keys interpolation kernel and the algorithm for estimation of optimal kernel parameters. The second part of this paper describes an experiment which was used to testing precision of the audio signal interpolation by 3P Keys kernel. Interpolation was performed over the audio signals from the Test base. The Test base consists of Sine and Audio test signals. The precision of interpolation was performed by using the MSE. The results are displayed in tables and graphs. A detailed comparative analysis showed the superiority of the suggested 3P Keys interpolation kernel over the 1P Keys and 2P Keys interpolation kernels described in related literature.

Keywords: interpolation, interpolation kernel, optimal parameter, audio processing, convolution.

1. INTRODUCTION

Many information systems require audio signals processing. Therefore, there has been made a large number of algorithms which process signals in time and frequency domain. [1]. With discrete electrical signals, there is often a need for real-time interpolation (loss of sample, change of measurement frequency, etc.). Problems with estimation of parameters are quite actual such as frequency and phase, where interpolation is required. Convolutional interpolation is suitable for real-time operation, because it works with a lower-order kernel. For the needs of convolutional interpolation, a large number of kernel have been developed [2]. In paper [3] Keys has suggested parametric convolutional kernel. By introducing the parametric kernels, there is a possibility to affect precision and efficiency interpolation algorithms. The results of the Keys kernel apply at estimation of fundamental frequencies of the speech signal are shown in paper [4]. By choosing the optimal parameter of kernel, it is possible to increase precision of Keys interpolation kernel. There are many algorithms for optimization of kernel parameters. Optimization is performed in: a) time and b) spectral domains. In [4] appliance of parametric kernels is suggested at image

processing and the algorithm for assessment of optimal parameter of kernel is suggested α_{opt} . Optimal value at image processing is $\alpha = -0.5$.

In paper [5] is shown two – parametric (α , β) interpolation kernel which has been constructed by expanding 1P Keys kernel [3]. This kernel was named 2P Keys kernel. In paper [6] was decided parameter's optimal value for estimation of fundamental frequency of the speech signal ($\alpha_{opt} = 0.1$, $\beta_{opt} = 0.2975$). In paper [7], an algorithm for optimizing the parameters of the 2P Keys kernel in the spectral domain was presented. In paper [8], the construction of a 3P kernel (α , β , γ) is based on the 1P Keys kernel [3] and was presented. This kernel is called 3P Keys. The optimal values of the 3P Keys kernel parameters were determined for estimating the fundamental frequency of the speech signal ($\alpha_{opt} = -1.7$, $\beta_{opt} = -4.7$, $\gamma_{opt} = -3.8$).

In this paper, by various experiments, the optimal parameters (α_{opt} , β_{opt} , γ_{opt}), for the Audio and Sine signals interpolation, are determined. The interpolation was done by applying 1P, 2P and 3P Keys kernel. After that, the MSE interpolation error of Audio and Sine test signal was calculated [4]. Audio test signals were obtained by recording G tones (G1 - G7) by Stainway B concert piano. The recording was realized at Iowa University

Acoustics Laboratory
(<http://theremin.music.uiowa.edu/MIS.html>)
with $f_s = 44.1$ kHz i 16 bps. Sine test signals were created with fundamental frequencies f_0 which respond to the basic tone frequencies G1 - G7. The Sine test signal is superimposed on a series of n sinusoidal signals amplitude a_n and frequency $n \cdot f_0$, where is $n = 2, \dots, 10$. Amplitudes were determined by random law in range 0 - 1V. The results of the MSE were displayed by graphics and tables. Comparative analysis data results were done by MSE for 1P, 2P and 3P Keys kernel. Therefore, the efficiency of 3P Keys kernel were estimated.

This paper is organized in the following way: Section 2 describes analytic form of 1P, 2P and 3P Keys kernels. Section 3 shows the Algorithm for the estimation of the optimal parameter values of 3P Keys kernel. Experimental results and analysis are provided in Section 4. The concluding remarks are made in Section 5.

2. KEYS' KERNELS

2.1 1P Keys Kernel

Paper [3] defines the 1P cubic interpolation kernel (1P Keys) as follows:

$$r(x) = \begin{cases} (\alpha + 2)|x|^3 - (\alpha + 3)|x|^2 + 1, & 0 \leq |x| \leq 1 \\ \alpha|x|^3 - 5\alpha|x|^2 + 8\alpha|x| - 4\alpha, & 1 < |x| \leq 2 \\ 0, & |x| > 2 \end{cases}, \quad (1)$$

where α is the kernel parameter. The length of 1P Keys kernel is $L = 4$.

2.2 2P Keys kernel

Paper [5] suggests a modification of 1P Keys kernel by introducing a second parameter and in this way, the 2P Keys kernel, of length $L = 6$, was formed. The analytical form of the 2P Keys kernel is:

$$r(x) = \begin{cases} (\alpha - \beta + 2)|x|^3 - (\alpha - \beta + 3)|x|^2 + 1, & 0 \leq |x| \leq 1 \\ \alpha|x|^3 - (5\alpha - \beta)|x|^2 + (8\alpha - 3\beta)|x| - (4\alpha - 2\beta), & 1 < |x| \leq 2 \\ \beta|x|^3 - 8\beta|x|^2 + 21\beta|x| - 18\beta, & 2 < |x| \leq 3 \\ 0, & |x| > 3 \end{cases}, \quad (2)$$

where α and β are kernel parameters. For $\beta = 0$ the 1P Keys kernel is obtained (Eq. (1)).

2.3 3P Keys kernel

Paper [8] defines the 3P interpolation kernel as follows:

$$r(x) = \begin{cases} (\alpha - \beta + \gamma + 2)|x|^3 + (-\alpha + \beta - \gamma - 3)|x|^2 + 1, & 0 \leq |x| \leq 1 \\ \alpha|x|^3 + (-5\alpha - \beta - \gamma)|x|^2 + (8\alpha - 3\beta + 3\gamma)|x| + (-4\alpha + 2\beta - 2\gamma), & 1 < |x| \leq 2 \\ \beta|x|^3 + (-8\beta + \gamma)|x|^2 + (21\beta - 5\gamma)|x| + (-18\beta + 6\gamma), & 2 < |x| \leq 3 \\ \gamma|x|^3 - 11\gamma|x|^2 + 40\gamma|x| - 48, & 3 < |x| \leq 4 \\ 0, & |x| > 4 \end{cases}, \quad (3)$$

where α , β and γ are kernel parameters. When $\gamma = 0$ the 2P Keys kernel is obtained [5]. When $\gamma = 0$ and $\beta = 0$ the 1P Keys kernel is obtained [3]. This is why the kernel suggested in the paper [8] was named 3P Keys kernel. The length of 3P Keys kernel is $L = 8$. Figure 1 shows 3P Keys kernel for different parameter values α , β and γ .

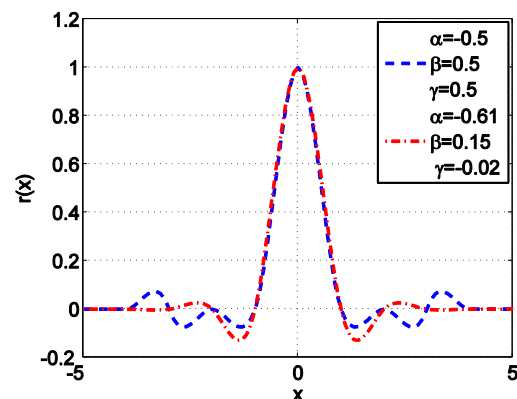


Fig. 1. 3P Keys kernel.

3P Keys kernel can be decomposed to a sum of components:

$$r(x) = r_0(x) + \alpha r_1(x) + \beta r_2(x) + \gamma r_3(x), \quad (4)$$

$$r_0(x) = \begin{cases} 2|x|^3 - 3|x|^2 + 1, & 0 \leq |x| \leq 1 \\ 0, & |x| > 1 \end{cases}, \quad (5)$$

$$r_1(x) = \begin{cases} |x|^3 - |x|^2, & 0 \leq |x| \leq 1 \\ |x|^3 - 5|x|^2 + 8|x| - 4, & 1 < |x| \leq 2 \\ 0, & |x| > 2 \end{cases}, \quad (6)$$

$$r_2(x) = \begin{cases} -|x|^3 + |x|^2, & 0 \leq |x| \leq 1 \\ |x|^2 - 3|x| + 2, & 1 < |x| \leq 2 \\ |x|^3 - 8|x|^2 + 21|x| - 18, & 2 < |x| \leq 3 \\ 0, & |x| > 3 \end{cases}, \quad (7)$$

and

$$r_3(x) = \begin{cases} |x|^3 - |x|^2, & 0 \leq |x| \leq 1 \\ -|x|^2 + 3|x| - 2, & 1 < |x| \leq 2 \\ |x|^2 - 5|x| + 6, & 2 < |x| \leq 3 \\ |x|^3 - 11|x|^2 + 40|x| - 48, & 3 < |x| \leq 4 \\ 0, & |x| > 4 \end{cases}. \quad (8)$$

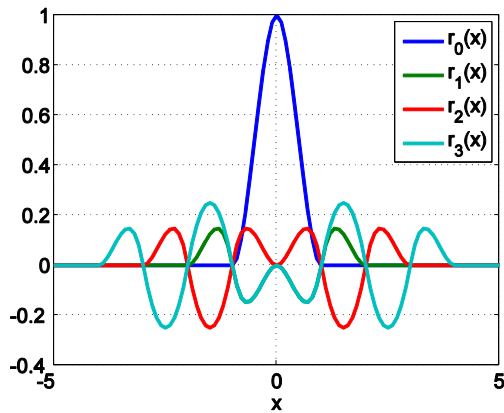


Fig. 2. 3P Keys kernel components.

3. ALGORITHM FOR THE ESTIMATION OF THE OPTIMAL KERNEL PARAMETERS

Algorithm for the estimation of the optimal parameter values $(\alpha_{opt}, \beta_{opt}, \gamma_{opt})$ for interpolation kernel r is realized in the following steps:

Input: X - test signal, N - signal length, L - kernel length, r_0 , r_1 , r_2 and r_3 - kernel components, α_{min} , $\Delta\alpha$, α_{max} , β_{min} , $\Delta\beta$, β_{max} , γ_{min} , $\Delta\gamma$, γ_{max}

Output: α_{opt} , β_{opt} , γ_{opt} .

FOR $\gamma = \gamma_{min} : \Delta\gamma : \gamma_{max}$

FOR $\beta = \beta_{min} : \Delta\beta : \beta_{max}$

FOR $\alpha = \alpha_{min} : \Delta\alpha : \alpha_{max}$

Step 1: Construction of the kernel:

$$r = r_0 + \alpha r_1 + \beta r_2 + \gamma r_3. \quad (9)$$

Step 2: The length of interpolation block is:

$$M = 2 \cdot L - 1.$$

FOR $I = 1 : N - M + 1$

Step 3: Selecting the I -th block:

$$X_I = X(1:I:M-1).$$

Step 4: Estimation of \hat{x}_I by applying PCC:

$$\hat{x}_I = X_I[1:2:M] \otimes r,$$

where the symbol \otimes stands for convolution.

Step 5: Estimation error:

$$e(I) = X_I(L) - \hat{x}_I.$$

END I

Step 6: Mean square error of estimation of 1P kernel:

$$MSE_\alpha(\alpha) = \frac{1}{N - M + 1} \sum_{k=1}^{N-M+1} |e(k)|^2. \quad (10)$$

END α

Step 7: Mean square error of estimation of 2P kernel:

$$MSE_{\alpha\beta}(\beta) = MSE_\alpha. \quad (11)$$

END β

Step 8: Mean square error of estimation of 3P kernel:

$$MSE_{\alpha\beta\gamma}(\gamma) = MSE_{\alpha\beta}, \quad (12)$$

END γ

Step 9: Optimal values of 3P kernel parameters:

$$(\alpha_{opt}, \beta_{opt}, \gamma_{opt}) = \arg \min_{\alpha, \beta, \gamma} (MSE_{\alpha\beta\gamma}). \quad (13)$$

4. EXPERIMENTAL RESULTS AND ANALYSIS

4.1 The Experiment

A Test signal base was formed consisting of Sine and Audio test signals. Then, the interpolation of test signals was conducted by using 1P, 2P and 3P Keys kernels and the precision of interpolation was estimated. Interpolation was carried out by using the algorithm described in Section 3. The precision of interpolation was shown by using MSE. After that, a detailed comparative analysis of the results was conducted.

4.2 The Base

The base consists of: a) Sine and b) Audio test signals. A Sine test signal is defined as:

$$s(t) = \sum_{i=1}^K a_i \sin(2\pi f_0 t), \quad (14)$$

where f_0 is the fundamental frequency, a_i the amplitude of i -th harmonic and K is the

number of harmonics. A base of Sine test signals was created which have fundamental frequencies that correspond to tones G1 (SinG1, $f_0 = 49\text{Hz}$), G2 (SinG2, $f_0 = 98\text{Hz}$), G3 (SinG3, $f_0 = 196\text{Hz}$), G4 (SinG4, $f_0 = 392\text{Hz}$), G5 (SinG5, $f_0 = 783.99\text{Hz}$), G6 (SinG6, $f_0 = 1567.98\text{Hz}$), G7 (SinG7, $f_0 = 3135.96\text{Hz}$) and to parameters $K = 10$, and amplitudes $a_i = \{0.98, 0.34, 0.2, 0.2, 0.34, 0.18, 0.19, 0.2, 0.34, 0.1\}$. The Sine test signal which corresponds to tone G2, $f_0 = 98\text{ Hz}$, is shown in: a) Fig. 3.a (time domain) and b) Fig.3.b (spectral domain). Audio test signals were acquired by recording G tones (G1 - G7) on a Steinway B concert piano. The recording was performed in the acoustics laboratory of Iowa University. The test signals were archived on the hard disc in the form of **wav** files. The recording was carried out by using $f_s = 44.1\text{ kHz}$ and 16 bits. Audio test signal of the tone G2 ($f_0 = 98\text{ Hz}$), and is presented in: a) Fig. 4.a (time domain) and b) Fig.4.b (spectral domain).

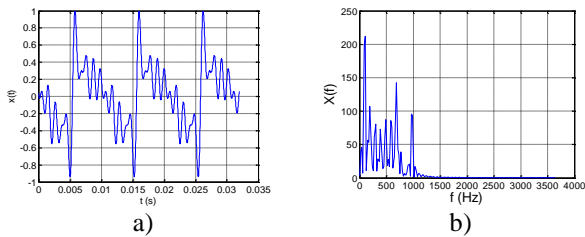


Fig. 3. Sine test signal (SinG2, $f_0 = 98\text{ Hz}$): a) time and b) spectral domain.

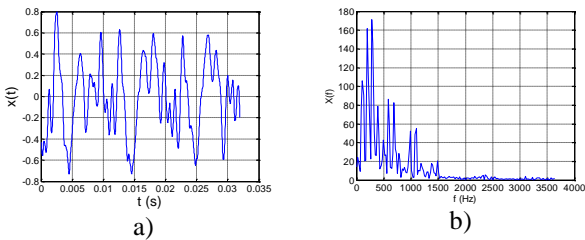


Fig. 4. Audio test signal (tone G2, $f_0 = 98\text{ Hz}$): a) time and b) spectral domain.

4.3 Results

Fig. 5 – Fig. 7 graphically show the results of MSE obtained by the application of algorithm for estimating the interpolation kernel parameters (Section 3). Fig. 5 presents the mean square error of estimate MSE_α (Eq.

(10)) by using 1P Keys kernel for: a) Sine test signal (SinG2, $f_0 = 98\text{ Hz}$) and b) Audio test signal (tone G2, $f_0 = 98\text{ Hz}$). Fig. 6.a shows the $MSE_{\alpha\beta}$ (Eq. (11)) for 2P Keys kernel for Sine test signal (SinG2). Fig. 6.b shows the position of the minimum MSE for Keys 1P (point A) and Keys 2P (point B) in the plane $(\alpha\beta)$. Fig. 7.a shows $MSE_{\alpha\beta}$ (Eq. (11)) for 2P Keys kernel for Audio test signal (G2). Fig. 7.b presents the minimum positions of MSE for Keys 1P (point A) and Keys 2P (point B) in the plane $(\alpha\beta)$. Fig. 8 shows the trajectory of minimal error (MSE_{\sin_3P}) for Sine test signals SinG1 - SinG7. Fig. 9 shows the trajectory of minimal error ($MSE_{\text{Eas_3P}}$) for Audio test signals G1 - G7.

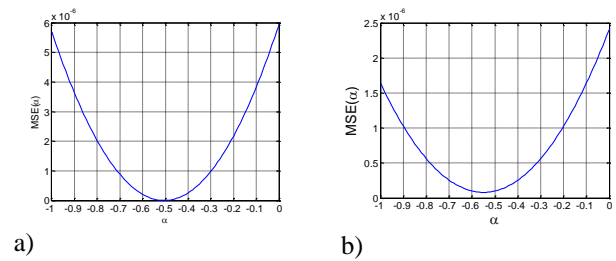


Fig. 5. MSE_α for: a) Sine test signal ($f_0 = 98\text{Hz}$) and b) Audio test signal (tone G2, $f_0 = 98\text{Hz}$)

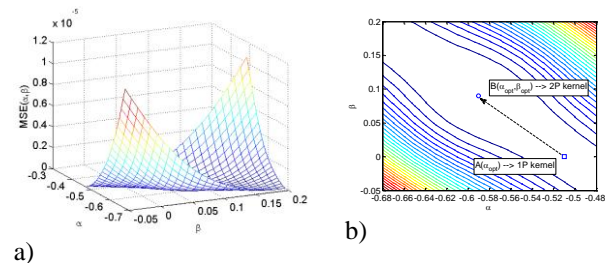


Fig. 6. a) Mean square error $MSE_{\alpha\beta}$ for Sine test signal (Fig. 3.a) and b) position of the minimum MSE (point A(α_{opt} , β_{opt})) in plane $(\alpha\beta)$.

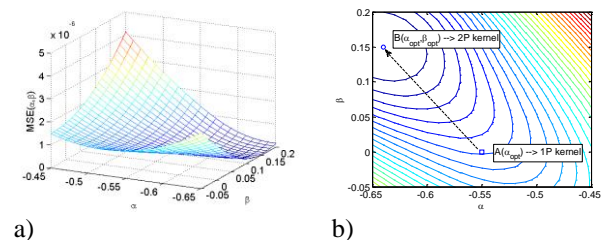


Fig. 7. a) Mean square error $MSE_{\alpha\beta}$ for Audio test signal (Fig. 4.a) and b) position of the minimum MSE (point A(α_{opt} , β_{opt})) in plane $(\alpha\beta)$.

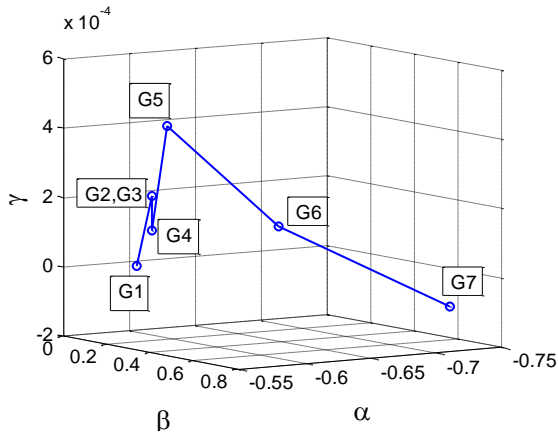


Fig. 8. Trajectory of minimal error ($MSE_{\sin_{3P}}$) for Sine test signals SinG1 - SinG7.

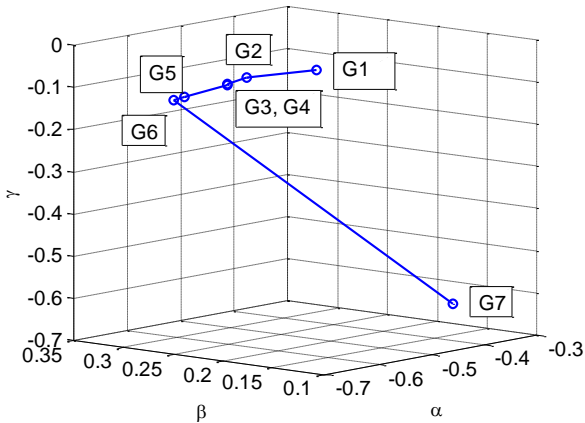


Fig. 9. Trajectory of minimal error ($MSE_{as_{3P}}$) for Audio test signals G1 - G7.

Optimal values of kernel parameters and minimal values of MSE for Sine signals are shown in: a) Table 1 (1P Keys, Eq. (10)), b) Table 2 (2P Keys, Eq. (11)) and c) Table 3 (3P Keys, Eq. (12)). Optimal values of kernel parameters and minimal values of MSE for Audio signals are shown in: a) Table 4 (1P Keys, Eq. (10)), b) Table 5 (2P Keys, Eq. (11)) and c) Table 6 (3PKeys, Eq. (12)). All of the tables show the mean values of optimal parameters and minimal values of MSE.

Table 1. Minimal MSE for 1P Keys kernel for a Sine test signal.

Ton	$\alpha_{opt_sin_1P}$	MSE_{sin_1P}
SinG1	-0.50	$1.8923 * 10^{-14}$
SinG2	-0.51	$5.5837 * 10^{-10}$
SinG3	-0.51	$3.5330 * 10^{-10}$
SinG4	-0.53	$2.0597 * 10^{-8}$
SinG5	-0.50	$1.6222 * 10^{-9}$
SinG6	-0.52	$7.1682 * 10^{-10}$
SinG7	-0.58	$1.2600 * 10^{-7}$
	$\overline{\alpha_{opt_sin_1P}}$	$\overline{MSE_{sin_1P}}$
	-0.5214	$2.1407 * 10^{-8}$

Table 2. Minimal MSE for 2P Keys kernel for a Sine test signal.

Ton	$\alpha_{opt_sin_2P}$	$\beta_{opt_sin_2P}$	MSE_{sin_2P}
SinG1	-0.59	0.09	$3.1737 * 10^{-17}$
SinG2	-0.59	0.09	$1.9511 * 10^{-12}$
SinG3	-0.60	0.10	$3.0627 * 10^{-12}$
SinG4	-0.60	0.10	$1.2895 * 10^{-11}$
SinG5	-0.59	0.09	$4.0466 * 10^{-12}$
SinG6	-0.68	0.20	$5.9297 * 10^{-12}$
SinG7	-0.74	0.62	$2.33382 * 10^{-11}$
	$\overline{\alpha_{opt_sin_2P}}$	$\overline{\beta_{opt_sin_2P}}$	$\overline{MSE_{sin_2P}}$
	-0.6271	0.1843	$7.3176 * 10^{-12}$

Table 3. Minimal MSE for 3P Keys kernel for a Sine test signal.

Ton	$\alpha_{opt_sin_3P}$	$\beta_{opt_sin_3P}$	$\gamma_{opt_sin_3P}$	MSE_{sin_3P}
SinG1	-0.59	0.09	0	$3.1737 * 10^{-17}$
SinG2	-0.60	0.10	$2 * 10^{-4}$	$2.9565 * 10^{-13}$
SinG3	-0.60	0.10	$2 * 10^{-4}$	$1.8452 * 10^{-13}$
SinG4	-0.60	0.10	$1 * 10^{-4}$	$1.2337 * 10^{-11}$
SinG5	-0.61	0.11	$4 * 10^{-4}$	$2.2423 * 10^{-14}$
SinG6	-0.68	0.20	$1 * 10^{-4}$	$5.626 * 10^{-12}$
SinG7	-0.74	0.62	$-1 * 10^{-4}$	$1.2838 * 10^{-12}$
	$\overline{\alpha_{opt_sin_3P}}$	$\overline{\beta_{opt_sin_3P}}$	$\overline{\gamma_{opt_sin_3P}}$	$\overline{MSE_{sin_3P}}$
	-0.6314	0.1886	$1.285 * 10^{-4}$	$2.8213 * 10^{-12}$

Table 4. Minimal MSE for 1P Keys kernel for an Audio test signal.

Ton	$\alpha_{opt_as_1P}$	MSE_{as_1P}
G1	-0.54	$2.2631 * 10^{-8}$
G2	-0.55	$8.1281 * 10^{-8}$
G3	-0.62	$1.7879 * 10^{-7}$
G4	-0.65	$9.7925 * 10^{-7}$
G5	-0.62	$1.0081 * 10^{-6}$
G6	-0.62	$5.2395 * 10^{-6}$
G7	-0.64	$2.5157 * 10^{-6}$
	$\overline{\alpha_{opt_as_1P}}$	$\overline{MSE_{as_1P}}$
	-0.6057	$1.4322 * 10^{-6}$

Table 5. Minimal MSE for 2P Keys kernel for an Audio test signal.

Ton	$\alpha_{opt_as_2P}$	$\beta_{opt_as_2P}$	MSE_{as_2P}
G1	-0.62	0.13	$1.8908 * 10^{-8}$
G2	-0.64	0.15	$4.0060 * 10^{-8}$
G3	-0.66	0.19	$3.9157 * 10^{-8}$
G4	-0.66	0.22	$1.5187 * 10^{-7}$
G5	-0.70	0.25	$3.2031 * 10^{-7}$
G6	-0.73	0.31	$1.6017 * 10^{-6}$
G7	-0.66	0.32	$1.1134 * 10^{-6}$
	$\overline{\alpha_{opt_as_2P}}$	$\overline{\beta_{opt_as_2P}}$	$\overline{MSE_{as_2P}}$
	-0.6671	0.2243	$4.6934 * 10^{-7}$

Table 6. Minimal MSE for 3P Keys kernel for an Audio test signal.

Ton	$\alpha_{opt_as_3P}$	$\beta_{opt_as_3P}$	$\gamma_{opt_as_3P}$	MSE_{as_3P}
G1	-0.62	0.15	0.0137	$1.8792 * 10^{-8}$
G2	-0.62	0.22	-0.0522	$3.4705 * 10^{-8}$
G3	-0.60	0.25	-0.825	$2.6165 * 10^{-8}$
G4	-0.60	0.25	-0.0855	$7.2428 * 10^{-8}$
G5	-0.55	0.32	-0.1476	$1.2647 * 10^{-7}$
G6	-0.57	0.32	-0.1495	$8.1891 * 10^{-7}$
G7	-0.42	0.12	-0.6057	$2.6207 * 10^{-7}$
	$\overline{\alpha_{opt_as_3P}}$	$\overline{\beta_{opt_as_3P}}$	$\overline{\gamma_{opt_as_3P}}$	$\overline{MSE_{as_3P}}$
	-0.5686	0.2329	-0.2645	$1.9422 * 10^{-7}$

4.4 Analysis of the results

Based on experimental results 1P Keys (Tbl. 1 and Tbl. 4), 2P Keys (Tbl. 2 and Tbl. 5) and 3P Keys kernel (Tbl. 3 and Tbl. 6) by their mutual comparison MSE shows that the application of 3P Keys kernel error is for:

a) Sine test signal: is i) $1P \rightarrow \frac{\overline{MSE}_{\sin_{1P}}}{\overline{MSE}_{\sin_{3P}}} = 2.1407 \cdot 10^{-8} / 2.8213 \cdot 10^{-12} = 7.5876 \cdot 10^3$, $2P \rightarrow \frac{\overline{MSE}_{\sin_{2P}}}{\overline{MSE}_{\sin_{3P}}} = 7.3176 \cdot 10^{-12} / 2.8213 \cdot 10^{-12} = 2.5937$ times smaller.

b) Audio test signal: is i) $1P \rightarrow \frac{\overline{MSE}_{as_{1P}}}{\overline{MSE}_{as_{3P}}} = 1.4322 \cdot 10^{-6} / 1.9422 \cdot 10^{-7} = 7.374$, $2P \rightarrow \frac{\overline{MSE}_{as_{2P}}}{\overline{MSE}_{as_{3P}}} = 4.6934 \cdot 10^{-7} / 1.9422 \cdot 10^{-7} = 2.4166$ times smaller.

By comparison of MSE experimental results for Audio and Sine signals by 3P Keys kernel application (Tbl. 6 and Tbl. 3) was concluded that is $\frac{\overline{MSE}_{as_{3P}}}{\overline{MSE}_{\sin_{3P}}} = 1.9422 \cdot 10^{-7} / 2.8213 \cdot 10^{-12} = 6.8839 \cdot 10^4$ times smaller error with Sine test signal interpolation.

CONCLUSION

In this paper, the optimization of 3P Keys interpolation kernel parameters was conducted. Parameter optimization was conducted by experiments. The experiment in which the precision of Sine and Audio test signals interpolation was conducted. Interpolation precision was expressed via MSE. Detailed comparative analysis shows that 3P Keys kernel with experimentally determined optimal parameters is more precision than 1P Keys kernel and 2P Keys kernel.

Based on displayed results, it has been concluded that 3P Keys kernel is more superior than 1P Keys kernel and 2P Keys kernel and that the interpolation errors were numerically very

small. 3P Keys kernel with optimal parameters is suitable for implementation in convolutional interpolation for real-time regime.

REFERENCE

- [1] A. Klapuri, "Multiple fundamental frequency estimation based on harmonicity and spectral smoothness", IEEE Transactions On Audio, Speech, And Language Processing, Vol. 11, No. 6, pp. 804-816, 2003.
- [2] E. Meijering, M. Unser, "A Note on Cubic Convolution Interpolation", IEEE Transactions on Image Processing, Vol. 12, No. 4, pp. 447-479, 2003.
- [3] R. G. Keys, "Cubic convolution interpolation for digital image processing", IEEE Trans. Acoust. Speech, & Signal Processing, vol. ASSP-29, pp. 1153-1160, Dec. 1981.
- [4] H. S. Pang, S.J. Baek, K.M. Sung, "Improved Fundamental Frequency Estimation Using Parametric Cubic Convolution", IEICE Trans. Fundamentals, vol. E83-A, No. 12, pp. 2747-2750, Dec. 2000.
- [5] R. Hanssen, R. Bamler, "Evaluation of Interpolation Kernels for SAR Interferometry", IEEE Transactions on Geoscience and Remote Sensing, vol. 37, no.1, pp. 318-321, Jan. 1999.
- [6] Z. Milivojević, D. Brodić, "Estimation Of The Fundamental Frequency Of The Real Speech Signal Compressed By MP3 Algorithm", Archives of Acoustics, Vol. 38. No. 3, pp. 363-373, 2013.
- [7] Z. Milivojević, N. Savić, D. Brodić, P. Rajković, "Optimization Parameters of Two Parameter Keys Kernel in the Spectral Domain", XV Internacional Scientific-Professional Symposium INFOTEH-Jahorina 2016.
- [8] Z. Milivojević, N. Savić, D. Brodić, "Three-Parametric Cubic Convolution Kernel For Estimating The Fundamental Frequency Of The Speech Signal", Computing and Informatics, Vol. 36, pp. 449-469, 2017.