UNITECH 2021 GABROVO

INTERNATIONAL SCIENTIFIC CONFERENCE
19-20 November 2021, GABROVO

GENERALIZATION OF A NEW THEORY OF FIR DIGITAL FILTERS INTENDED FOR IMPULSE SIGNALS

Perišić Đurđe[1], Miroslav Bojović[2], Željko Gavrić[1], Vanja Mišković[1]
[1] Faculty of Information Technology, Slobomir P University, Slobomir, BiH
[2] School of Electrical Engineering, University of Belgrade, Beograd, Serbia

Abstract
A new kind of FIR (Finite Impulse Response) digital filter is recently described, which is intended for the filtering of impulse periods. This filter is by nature a Frequency Locked Loop (FLL), which is based on the measurement and processing of the input periods only. In this article, the general equations for transfer functions of FLL of any order are derived, as well as the general Z transformations of the FLL outputs. These general equations enable fast and simple adapting FLL of any order to function as digital filter, using Mat-lab tools, dedicated for the design of the classical FIR digital filter. The application of these equations is demonstrated on the development of digital filter based on FLL of the fifth order. The filtering ability of the fifth-order FLL is demonstrated for one practical example.

Keywords: Digital circuits, Digital filters, PLL, FLL, Pulse circuits.

INTRODUCTION

It was shown in ref. [1] that, Mat-lab tools intended for classical FIR digital filters, can be used for frequency analysis of the Frequency Locked Loops (FLL), which are based on the measurement and processing of the input signal periods only (called period FLL). Using the theory and tools of classical FIR digital filters, it was demonstrated in ref. [2], how a new type of FIR digital filters can be designed from period FLL (called FLL digital filters). These new FLL digital filters (type FIR) are intended for the filtering of the pulse signal periods.

Ref. [2] points out the differences and similarities between amplitude processing in classical digital filters and period processing in FLLs. For the further development of FLL digital filters, it is of interest to define its difference equation of M-th order and to compare it with the same difference equation of the classical FIR digital filter. Since there is an inevitable overlap between the input and input periods of the same order, when processing periods, a time difference between the input and output periods τ was introduced in refs. [1] to [10], as a variable that shows how much this overlap is in the positive or negative direction. Time difference τ can always be easily translated into a phase difference between the input and output periods. In addition, as shown in ref. [1], the time difference τ also contains information carried by the input period. Time difference τ has a different transfer function and different filter characteristics compared to the output period, and provides an additional range of FLL applications. Therefore, in the generalization of the theory of FLL digital filters, the time difference τ should also be included as the second output variable, together with the output period.

In the papers [3] to [12], the period FLLs and period Phase Locked Loops (PLLs) were applied in the field of frequency averaging, phase shifting, time shifting, phase control, tracking, predicting, frequency synthesizers, noise rejection, frequency multipliers and the others. Besides refs. [1] and [2], they are also closely related to this paper. The rest of articles and books [13] to [18] in the references, are used as the theoretical base.

FLL DIGITAL FILTER OF M-th ORDER

A general case of an input signal Sin and an output signal Sop of period FLL is presented in Fig. 1. In comparison to the classical PLL and FLL, the input and output frequencies are changed by the input and output periods in Fig. 1, and the phase differences are changed by the time differences. The periods TI_k and TO_k, as well as the time difference, occur at discrete times t_k, t_{k+1}, t_{k+2},… t_{k+M}, t_{k+M+1}, which are defined by the falling edges of the pulses of Sop in Fig. 1. The first important difference

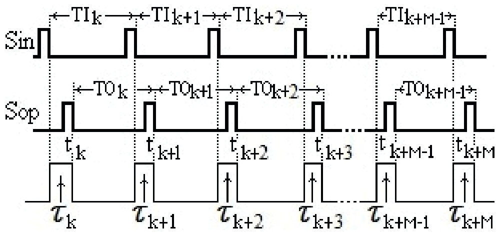

Fig. 1. *The time relations between the input and output variables of period FLL of M-th order.*

between the processing described by the classical digital filters and the processing of the period FLLs is the fact that instead of the amplitudes, the period FLL uses the input periods in the processing. Secondly, unlike the amplitudes which are sampled with the constant time intervals, the input and output periods TI_k and TO_k as well as time differences τ_k, are distributed in time in Fig. 1, so that every input period overlaps with the output period of the same order. Due to this distribution and overlapping in time, it is not possible to calculate, for instance, the output period TO_{k+1} as a function of TI_{k+1}, because the calculation of TO_{k+1} must be finished up to discrete time t_{k+1}, i.e. before the input period TI_{k+1} is expired, see Fig. 1. At discrete time t_{k+1} the realization of TO_{k+1} should start. In other word, in the real time applications, any output period can be calculated only using the previous input periods of the lower order. Taking this fact in account, the general difference equation which describes the period FLL of the M-th order, corresponding to Fig.1, is presented in eq. (1). Note that because of simplicity, all discrete times in brackets are changed with the corresponding index marks in eq. (1). For instance, TO(k+M) is changed with TO_{k+M}, TI(k+M-1) is changed with TI_{k+M-1} and so on. Equation (1a) is the shorted form of eq. (1). It comes out from eq. (1), that there are "M" system parameters of FLL of M-th order. These are b_1, b_2... b_M. If we want to complete the calculation of TO(k) with all system parameters, it is necessary "M" input periods. Another equation describing the natural relation between the input and output variables, which comes out from Fig. 1, is shown in eq. (2).

$$TO_{k+M} = b_1 TI_{k+M-1} + b_2 TI_{k+M-2} + \cdots + b_M TI_k \qquad (1)$$

$$TO_{k+M} = \sum_{i=1}^{M} b_i \cdot TI_{k+M-i} \qquad (1a)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \qquad (2)$$

It is very useful to compare eq. (1a) with the difference equation of classical FIR digital filters of the M-th order, which is shown in eq. (3), where the output of a digital filter y(k) is the sum of products of (M+1) filter coefficients b_0, b_1, b_2,... b_M and the corresponding samples of the input signal x(k-i). Note that the variable "k", represents the discrete time t_k when an amplitude of the input signal is sampled, measured and taken in calculation. According to eq. (3), there are (M+1) calculations of the outputs and (M+1) filter coefficients. Note that, to the case i=0 corresponds the coefficient b_0 in eq. (3), so that the calculation of y(k) can be performed using an input of the same order x(k). However, the parameter b_0 does not exist in eq. (1a).

$$y(k) = \sum_{i=0}^{M} b_i \cdot x(k-i) \qquad (3)$$

In order to find out the transfer functions of the period FLL it is necessary to determine the Z transforms of TO_{k+M} and τ_{k+1}, given by eqs. (1) and (2). The Z transform of eqs. (1) and (2) can be derived in two ways. The first way is to develop it directly from eqs. (1) and (2). The Z transformations of a M-th order FLL can also be performed from the multiple Z transformations of lower-order FLLs. We will apply the second approach, as the first approach would take up a lot of space. For FLL of the second order, ref. [1], TO(z) and $\tau(z)$ are shown in eqs. (4) and (5). Following the example of the second-order FLL, described in ref [1], the difference equations for third-order FLL are: $TO_{k+3}=b_1TI_{k+2}+b_2TI_{k+1}+b_3TI_k$ and $\tau_{k+1}=\tau_k+TO_k-TI_k$. Using the same procedure, like in Ref. [1], TO(z) and τ(z) for FLL of the third order are developed and shown in eqs. [6] and [7]. Based on eqs. (4), (5), (6) and (7), we can derive the Z transforms of the M-th order FLL, given in eqs. (8) and (9). The shorted form of eqs. (8) and (9) are presented in eqs. (10) and (11), where $H_{TO}(z)=TO(z)/TI(z)$ and $H_\tau(z)=\tau(z)/TI(z)$ are the transfer function of the M-th order FLL. The transfer functions are presented in eqs. (12) and (13). TO_0 and τ_0 in eqs. (4) to (11) are the initial conditions of the corresponding output periods and time differences. $R(z)=(TO_0 + z\tau_0)/(z-1)$ in eqs. (5), (7) and (9).

$$TO(z) = TI(z)\cdot(zb_1+b_2)/z^2+TO_0 \tag{4}$$

$$\tau(z) = TI(z)\cdot[-z+(b_1-1)]/z^2+R(z) \tag{5}$$

$$TO(z) = TI(z)\cdot(z^2b_1+zb_2+b_3)/z^3+TO_0 \tag{6}$$

$$\begin{aligned}\tau(z) = TI(z)\cdot[-z^2+z(b_1-1)\\ +(b_2+b_1-1)]/z^3+R(z)\end{aligned} \tag{7}$$

$$\begin{aligned}TO_M(z) = TI(z)\cdot(z^{M-1}b_1+z^{M-2}b_2+...\\ +zb_{M-1}+b_M)/z^M+TO_0\end{aligned} \tag{8}$$

$$\begin{aligned}&\tau_M(z) = TI(z)\cdot[-z^{M-1}+z^{M-2}(b_1-1)+\\ &z^{M-3}(b_2+b_1-1)+...+z(b_{M-2}+...+b_1-1)\\ &+(b_{M-1}+...+b_1-1)]/z^M+R(z)\end{aligned} \tag{9}$$

$$TO_M(z) = TI(z)\cdot H_{TO_M}(z)+TO_0 \tag{10}$$

$$\tau_M(z) = TI(z)\cdot H_{\tau_M}(z)+(TO_0+z\cdot\tau_0)/(z-1) \tag{11}$$

$$H_{TO_M}(z) = TO_M(z)/TI(z) = \sum_{i=1}^{M} b_i\cdot z^{M-i}/z^M \tag{12}$$

$$\begin{aligned}&H_{\tau_M}(z) = \{-z^{M-1}+\\ &\sum_{i=1}^{M-1} z^{M-1-i}\cdot[(\sum_{j=1}^{i} b_j)-1]\}/z^M\end{aligned} \tag{13}$$

Although eq. (13) looks complicated, it is together with eq. (12) very useful, because using them, we can easily derive Z transforms of the transfer functions of any order FLL, escaping long mathematical operations. According to refs. [1] and [2], in order to adapt a FLL to function as a digital filter, it is necessary to determine the transfer function of FLL. Using eqs. [12] and [13], let us determine the transfer functions for M=5, i.e. for the FIR FLL of the fifth order (FIR FLL_5), which is also described in ref. [2]. If we enter M=5 in eqs. (12) and (13), we will get the transform functions $H_{TO5}(z)$ and $H_{\tau5}(z)$ for FIR FLL_5, shown in eqs. (14) and (15). Using eqs. (10) and (11), the Z transform of the FIR FLL_5 outputs are determined and shown in eqs. (16) and (17). Comparing eqs. (14) and (15) with the corresponding $H_{TO5}(z)$ and $H_{\tau5}(z)$ in Ref. [2], we can see that they are identical. Comparing eqs. (16) and (17) with the corresponding TO(z) and τ(z) in Ref. [2], we can also see that they are identical, proving so the correctness of all previously presented maths in this article.

It follows from the previous conclusions that, using equations [10] and [11], we can determine the Z transformations TO(z) and τ(z) for a FLL of any order. Also, based on equations [12] and [13], we can determine the transfer functions $H_{TO}(z)$ and $H_\tau(z)$ for a FLL of any order. After determining these transfer functions, using the rules of Mat-lab, we can determine the corresponding vectors "b" and "a" used in Mat-lab for the FIR digital filters. In ref. [1], it is shown that based on these vectors “b” and “a”, detailed analyzes of a FLL in the frequency domain can be performed, using Mat-lab tools intended for the design and analysis of the FIR digital filters. This whole procedure of the design and analyses is also described in detail in ref. [2] for the fifth-order FLL.

$$H_{TO5}(z) = (z^4b_1+z^3b_2+z^2b_3+zb_4+b_5)/z^5 \tag{14}$$

$$\begin{aligned}&H_{\tau5}(z) = [-z^4+z^3(b_1-1)+z^2(b_2+b_1-1)+\\ &+z(b_3+b_2+b_1-1)+(b_4+b_3+b_2+b_1-1)]/z^5\end{aligned} \tag{15}$$

$$TO_5(z) = TI(z)\cdot H_{TO5}(z)+TO_0 \tag{16}$$

$$\tau_5(z) = TI(z)\cdot H_{\tau5}(z)+(TO_0+z\cdot\tau_0)/(z-1) \tag{17}$$

In the adapting a FLL to function as FIR digital filter in ref. [2], the system parameters of a FIR FLL_5 are changed by the coefficients od the classical FIR digital filter of the fourth order. It is not necessary to repeat the description of this procedure in this article. Instead of that, let us only demonstrate some filter characteristics of the low pass digital filter (type triangle) based on the fifth-order FLL. Let us suppose that the filter is defined by the cutoff frequency fg=2000 Hz and sampling frequency fs=14000 Hz. The input period TI_{k+1} is supposed to be $TI(k+1)=10+S_1(k)+S_2(k)$ [time units], where $S_1(k)=6\cdot\sin[2\pi/fs\cdot f_1\cdot k]$ and $S_2(k)=6\cdot\sin[2\pi/f_s\cdot f_2\cdot k]$. The values of frequencies f_1 and f_2 are $f_1=1500$ Hz and $f_2=4500$ Hz. The time unit [t.u.] can be, μsec, msec or any other, but assuming the same time units for all time variables. It was more suitable to use abbreviated [t.u.] in the text and to omit [t.u.] in a diagram. Following the same procedure of analyses, which is applied in ref. [2], the frequency spectrums of the input and output periods are determined and presented in Fig. 2 in the whole sample rate. It is visible in Fig. 2 that signal S_1 at 1500 Hz, is only slightly attenuated, since f_1 is less than cutoff frequency $f_g=2000$ Hz. At the same time signal S_2 at 4500 Hz is suppressed, because $f_2=4500$ Hz is greater than cutoff frequency $f_g=2000$ Hz. In other word, it belongs to the stop band of the low pass digital filter based on FLL_5. It can be also seen in Fig. 2, that the zero component at frequency close to zero is not attenuated, just like in ref. [2].

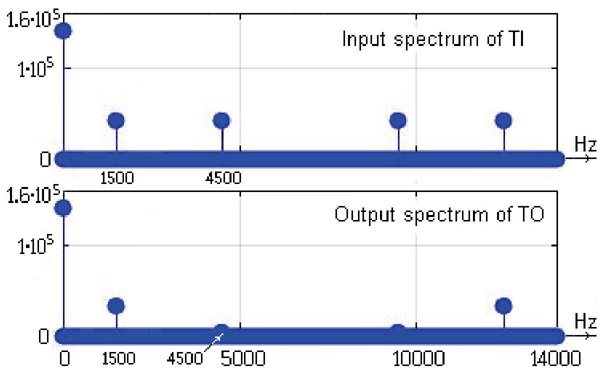

Fig. 2. *The input spectrum of TI and the output spectrum of TO.*

CONCLUSION

This article represents important contribution to the theory and application of new FIR FLL digital filters, which are intended for filtering of impulse signal period. Namely, in the procedure of filter development it was necessary, as the first step, to determine the transfer functions of the filter outputs (output period and time differences). This step precedes the application of the Mat-lab tools in which the zeros of a classical digital filter are assigned as zeros of the FLL in order for the FLL to function as a digital filter, refs. (1) and (2). However, the process of performing transfer functions can be very long and complex mathematical procedure. It takes long time, with very small probability to escape mistake. For a FLL of very high order, which is expected to be used in filtering, to perform this task without mistakes, becomes almost impossible.

Besides the transfer functions, since FLL functions in real time conditions, the development of Z transformations of the output variables (output period and time differences) of a FLL is necessary for the different kind of analyzes.

The derived equations enable fast, simple and secured derivations of the transfer functions and Z transformations of the outputs of a FLL of any order. At the same time, this step simplifies the design of any order FLL digital filter and almost reduces it to designing a classic digital filter. In this article, the application of these equations is demonstrated on the development of the FIR FLL digital filter of the fifth order.

Acknowledgements

This article was supported by the Ministry of Science and Technology of the Republic of Serbia within the project TR 32047.

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