INFINITE IMPULSE RESPONSE DIGITAL FILTER FOR PULSE SIGNAL PERIODS

Perišić Đurđe[1], Miroslav Bojović[2]
[1] *Faculty of Information Technology, Slobomir P University, Slobomir, BiH*
[2] *School of Electrical Engineering, University of Belgrade, Beograd, Serbia*

Abstract
This article describes a new kind of IIR (Infinite Impulse Response) digital filter, which is intended for the filtering of impulse periods. This filter is by nature a Frequency Locked Loop (FLL), which is based on the measurement and processing of the input and output periods. The paper shows how a digital filter can be designed from one FLL using the theory of classical IIR digital filters. All mathematical analyzes were performed using Z transformation and the theory of linear discrete systems. To illustrate this approach a Butterworth IIR digital filter is designed, which is based on a third-order FLL. The filtering ability of the third-order FLL is demonstrated for one practical example.

Keywords: Digital circuits, Digital filters, PLL, FLL, Pulse circuits, Linear discrete system.

INTRODUCTION

The field of the special type of Frequency Locked Loop (FLL), recently described in the literature, which is based on the measurement and processing of the pulse signal periods, has acquired its new, very important role in theory and practice. Namely, for a special choice of FLL parameters, this system becomes a digital filter like a classic digital filter, but such a system filters the pulse signal periods. It was shown in ref. [1] that, Mat-lab tools intended for the classical Finite Impulse Response (FIR) digital filters, can be used for the frequency analysis of FLL, which are based on the measurement and processing of the input signal periods only. Using the theory and tools of the classical FIR digital filters, it was demonstrated in ref. [2], how a new type of FIR digital filters can be designed from a period FLL. In refs. [1] and [2], the described FLLs process only the input signal periods. In accordance with the already known term from the theory of FIR digital filters, in which the filter processes only the samples of the input signal, we called the described filters FIR FLL digital filters.

This paper describes a methodology for designing a FLL digital filter that processes both, the input and output signal periods. In accordance with the adopted terminology from the theory of the Infinite Impulse Response (IIR) digital filters, let's call these systems the IIR FLL digital filters.

The refs. [3] to [11] describe the period FLLs and period Phase Locked Loops (PLLs), whose construction and functioning are the same as the FIR FLL and IIR FLL digital filters. However, for the different choice of parameters, they were applied in the field of frequency averaging, phase shifting, time shifting, phase control, tracking, predicting, frequency synthesizers, noise rejection, frequency multipliers and the others. The rest of articles and books [12] to [17] in the references, are used as the theoretical base.

DESIGN OF IIR FLL DIGITAL FILTER

For simplicity, the IIR FLL digital filter design will be demonstrated on the third-order FLL. The same principles that apply to the third-order FLL can be used on any higher-order FLL in order to design an IIR FLL digital filter. A general case of an input signal Sin and an output signal Sop of the third-order FLL is presented in Fig. 1. The periods TI_k and TO_k, as well as the time difference τ_k, occur at discrete times t_k, t_{k+1}, t_{k+2}, t_{k+3}…., which are defined by the falling edges of the pulses of

Sop in Fig. 1. The input and output periods TI_k and TO_k as well as the time differences τ_k, are distributed in time in Fig. 1, so that every input period overlaps with the output period of the same order. Due to this distribution and overlapping in time, it is not possible to calculate, for instance, the output period TO_{k+3} as a function of TI_{k+3}, because the calculation of TO_{k+3} must be finished up to discrete time t_{k+3}, i.e. before the input period TI_{k+3} is expired, see Fig. 1. At discrete time t_{k+3} the realization of TO_{k+3} should start. In other word, in the real time applications, any output period can be calculated only using the previous input periods of the lower order. Taking this fact in account, the general difference equation which describes the third-order FLL, corresponding to Fig.1, is presented in eq. (1), where b_1, b_2, b_3, a_1, a_2 and a_3 are the system parameters. Following the same idea, presented in ref. [2], that the FLL parameters should be changed by the coefficients of a classical digital filter, let us first compare eq. (1) with the difference equation of the second order FIR digital filter, which is presented in eq. (2). Note that y(k) is

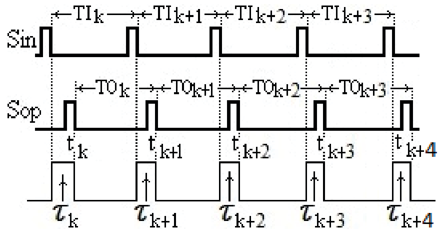

Fig. 1. *The time relations between the input and output periods and the time differences of the third-order FLL.*

$$TO_{k+3} = b_1 TI_{k+2} + b_2 TI_{k+1} + b_3 TI_k + a_1 TO_{k+2} + a_2 TO_{k+1} + a_3 TO_k \quad (1)$$

$$a_{0d} y_{k+2} + a_{1d} y_{k+1} + a_{2d} y_k = b_{0d} x_{k+2} + b_{1d} x_{k+1} + b_{2d} x_k \quad (2)$$

the calculated output and x(k) is the sampled input in digital form. All coefficients in eq. (2) have suffix “d”, to signify that they belong to the digital filter. Coefficient a_{0d} is always equal to one in a FIR digital filter. Note that in eq. (1), parameter a_0, which should be attached to TO_{k+3}, is also equal to one. Equation (2) corresponds to the form recognized by Mat-lab, while eq. (1) corresponds to a form which is suitable for the mathematical analysis of a real-time FLL. Including a_{0d} and a_0, we notice that eq. (1) contains 3 parameters "b" and 4 parameters "a", while equation (3) contains 3 coefficients b_d and 3 coefficients "a_d". In order to equalize the number of parameters with the number of coefficients, we must omit parameter a_3 in eq. (1). Equation (1) changes to modified form, given by eq. (3). Another equation describing the natural relation between the input and output periods and time differences τ_k, which comes out from Fig. 1, is shown in eq. (4). In order to compare eqs. (2) and (3), let us write eq. (2) in the form, similar to eq. (3), shown in eq. (5).

$$TO_{k+3} = b_1 TI_{k+2} + b_2 TI_{k+1} + b_3 TI_k + a_1 TO_{k+2} + a_2 TO_{k+1} \quad (3)$$

$$\tau_{k+1} = \tau_k + TO_k - TI_k \quad (4)$$

$$y_{k+2} = b_{0d} x_{k+2} + b_{1d} x_{k+1} + b_{2d} x_k - a_{1d} y_{k+1} - a_{2d} y_k \quad (5)$$

To design an IIR FLL digital filter of the third order (IIR FLL_3) using the Mat-lab tools, we need to find the Z transforms of the transfer functions of the described FLL using equations (3) and (4). The Z transforms of eqs. (3) and (4) are presented in eqs. (6) and (7). In eqs. (6) and (7), TO_0, TI_0 and τ_0 are the initial conditions of the variables TO_k, TI_k and τ_k. Based on eq. (3), for k=-2, $TO_1=b_1TI_0+ a_1TO_0$ and for k=-1, $TO_2=b_1TI_1+b_2TI_0+a_1TO_1+a_2TO_0$. Using the given expressions and eq. (6), TO(z) is calculated and shown in eq. (8), where $R(z)=z^3TO_0/(z^3- z^2a_1- za_2)$. It is now of interest to investigate under which conditions this IIR FLL_3 is the stable system. To do that, let us suppose that the step input is TI(k) = TI = constant. Substituting the Z transform of TI(k) i.e. $TI(z) = TI\cdot z/(z-1)$ into eq. (8) and using the final value theorem, it is possible to find the final value of the output period TO_∞, which IIR FLL_3 reaches in the stable state. We can calculate TO_∞ = lim TO(k) if $k\to\infty$, using TO(z). This is shown in eq. (9). It comes out from eq. (9), that TO_∞=TI if eq. (10) is satisfied. Changing TO(z) given by eq. (8) into eq. (7), $\tau(z)$ is calculated and shown in eq.

(11), where $S_{ab}= b_2 + b_1 + a_2 + a_1-1$. Based on eqs. (8) and (11), we can define two transfer functions $H_{TO}(z)$ and $H\tau(z)= \tau(z)/TI(z)$, shown in eqs. (12) an (13).

$$z^3TO(z)-z^3TO_0-z^2TO_1-zTO_2=b_1[z^2TI(z)$$
$$-z^2TI_0-zTI_1]+b_2[zTI(z)-zTI_0]+b_3TI(z)+ \quad (6)$$
$$a_1[z^2TO(z)-z^2TO_0-zTO_1]+a_2[zTO(z)-zTO_0]$$

$$z\tau(z)-z\tau_0=\tau(z)+TO(z)-TI(z) \quad (7)$$

$$TO(z)=TI(z)\frac{z^2b_1+zb_2+b_3}{z^3-z^2a_1-za_2}+R(z) \quad (8)$$

$$TO_\infty=\lim[(z-1)TO(z)]_{z\to 1}=TI\frac{b_1+b_2+b_3}{1-a_1-a_2} \quad (9)$$

$$b_1+b_2+b_3+a_1+a_2=1 \quad (10)$$

$$\tau(z)=TI(z)\frac{-z^2+z(b_2+b_1-1)+S_{ab}}{z^3-z^2a_1-za_2}+\frac{R(z)+z\tau_0}{z-1} \quad (11)$$

$$H_{TO}(z)=\frac{TO(z)}{TI(z)}=\frac{z^2b_1+zb_2+b_3}{z^3-z^2a_1-za_2} \quad (12)$$

$$H_\tau(z)=\frac{-z^2+z(b_2+b_1-1)+b_2+b_1+a_2+a_1-1}{z^3-z^2a_1-za_2} \quad (13)$$

Modified eq. (3) of FLL is structurally the same like eq. (5) and the next step is to simply change parameters in eq. (3) with the corresponding coefficients of the digital filter so that $b_1= b_{0d}$, $b_2= b_{1d}$, $b_3= b_{2d}$, $a_1= -a_{1d}$ and $a_2= -a_{2d}$. The transfer function $H_{TO}(z)$, eq. (12) turns to eq. (14). The transfer function of the digital filter $H_{DF}(z)$, according to eq. (2) or eq. (5) is shown in eq. (15). The transfer functions $H_{TO}(z)$ and $H_{DT}(z)$ cover the same zeros and poles, but the difference between them is in their denominators. Namely, their relation can be expressed as $H_{TO}(z)= H_{DF}(z)\cdot z^{-1}$. This means that the magnitudes of the frequency responses of $H_{TO}(z)$ and $H_{DF}(z)$ will be the same. But due to one step delay, which refers to factor "z^{-1}", IIR FLL_3 will introduce an additional delay of -π [rad] on the output signal, in relation to the phase which the digital filter makes on its output signal. Based on the Mat-lab rules for definitions of vector "b" and "a" of the IIR digital filters, we can define vectors b_{DF} and b_{TO}, as well as vectors a_{DF} and a_{TO}, using the transfer functions $H_{DF}(z)$ and $H_{TO}(z)$, eqs. (16), (17) and (18). If we change $b_1= b_{0d}$, $b_2= b_{1d}$, $b_3= b_{2d}$, $a_1= -a_{1d}$ and $a_2= -a_{2d}$ in eq. (13), we can determine vectors b_τ and a_τ, which correspond to eq. (13). Vectors b_τ and a_τ are shown in eqs. (19) and (20). All of vectors are necessary for the frequency analyses of the described IIR FLL_3 and the digital filter, using Mat-lab tools intended for the IIR digital filters.

$$H_{TO}(z)=\frac{TO(z)}{TI(z)}=\frac{z^2b_{0d}+zb_{1d}+b_{2d}}{z^2+za_{1d}+a_{2d}}\cdot z^{-1} \quad (14)$$

$$H_{DF}(z)=\frac{y(z)}{x(z)}=\frac{z^2b_{0d}+zb_{1d}+b_{2d}}{z^2+za_{1d}+a_{2d}} \quad (15)$$

$$b_{DF}=[b_{0d}\ \ b_{1d}\ \ b_{2d}] \quad (16)$$

$$b_{TO}=[0\ \ b_{0d}\ \ b_{1d}\ \ b_{2d}]=[0\ \ b_{DF}] \quad (17)$$

$$a_{DF}=a_{TO}=[1\ \ a_{1d}\ \ a_{2d}] \quad (18)$$

$$b_\tau=[0\ \ -1\ \ (b_{2d}+b_{1d}-1)\ \ (b_{2d}+b_{1d}-a_{2d}-a_{1d}-1)] \quad (19)$$

$$a_\tau=a_{DF}=a_{TO}=[1\ \ a_{1d}\ \ a_{2d}] \quad (20)$$

After we developed vectors "a" and "b", based on the transfer functions of the IIR FLL_3 outputs TO and τ, the further procedure of frequency analysis of these outputs can be performed in a completely identical way. In the following text, we will give the emphasis to the design and analysis of the filter characteristics of IIR FLL_3 using output TO and comparing it with the corresponding digital filter. In order to design an IRR FLL_3, we have to first design the corresponding IIR digital filter of the second order (IIR DF_2). Let us design Butterworth low pass IIR DF_2, defined by the cutoff frequency fg=2000 Hz and sampling frequency fs=10000 Hz. Using Mat-lab command [b_{DF}, a_{DF}] = butter (N, f_n), where the filter order N=2 and $f_n=f_g/(f_s/2)$, we can get vectors b_{DF}=[0.1867 0.3734 0.1867] and a_{DF}=[1 -0.4629 0.2097]. Note that eq. (10) is satisfied, if we change b_1=0.1867, b_2=0.3734, b_3=0.1867, a_1=-(-0.4629) and a_2=-0.2097. This means that after changing the parameters with the coefficients of IIR DF_2, IIR FLL_3 will stay stable. Since vectors b_{DF} and a_{DF} are now determined, b_{TO} and a_{TO} are defined by eqs. [17] and [18]. Based on these vectors and using Mat-lab commands freqz (b_{TO}, a_{TO}, 1024, fs) and freqz (b_{DF}, a_{DF}, 1024, fs), the frequency responses of IRR FLL_3 and IRR DF_2, are determined and presented in Fig.

2 for the half of the sample rate. It can be seen that the magnitudes of the IIR DF_2 and IIR FLL_3 are identical. Since both of IRR FLL_3 and IRR DF_2 are the IIR digital filters, no one of their phases is linear, but for the half of the sample rate, the phase of IIR FLL_3 is -360° and the phase of IIR DF_2 is -180°. The phases which two systems introduced into the output signals differ for expected -180°, for the half of the sample rate. This proves that the adaptation of the third-order FLL, with the aim of functioning as a second-order IIR digital filter, has been successfully realized.

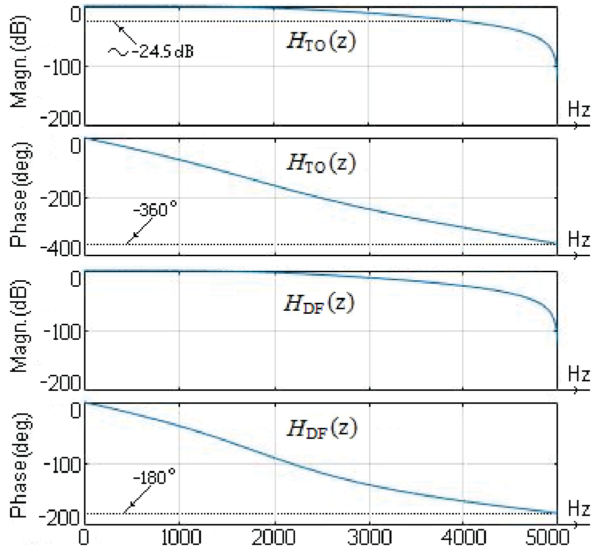

Fig. 2. *Magnitudes and phases of the frequency responses of $H_{TO}(z)$ and $H_{DF}(z)$.*

Let us demonstrate the filter characteristics of the Butterworth low pass digital filter based on the designed third-order FLL. Suppose that the input period TI_{k+1} is defined as $TI(k+1)= 6+S_1(k)+ S_2(k)$ [time units], where $S_1(k)= 5\cdot\sin[2\pi/fs\cdot f_1\cdot k]$ and $S_2(k)=5\cdot\sin[2\pi/f_s\cdot f_2\cdot k]$. The input periods are continuously changing under effects of two sinusoidal signals S_1 and S_2. Suppose that the values of frequencies are f_1=1000 Hz and f_2=4000 Hz. Note that the frequency f_1 is less than the cutoff frequency f_g=2000 Hz and the frequency f_2 is greater than f_g. The time unit [t.u.] can be, μsec, msec or any other, but assuming the same time units for all time variables. It was more suitable to omit [t.u.] in the diagrams. The first step in this presentation is to form vector TI of 10000 values of TI, using the above equation for TI_{k+1}. Based on the vector TI, the output period vector TO = filter (b_{TO}, a_{TO}, TI) is determined. This vector can also be formed on the basis of eqs. (3) and (4). After that, using the "fft" command, the input and output vectors of IIR FLL_3 are formed as X = fft (TI) and Y = fft (TO). Finally, using the command "stem", stem (abs (X)) and stem (abs (Y)), the spectrums of the input and output periods are presented in Fig. 3. These spectrums present the absolute values of the amplitudes, covering the whole sample rate. They appear as positive values in the symmetric second half of the sample rate. It is visible in Fig. 3 that signal S_1 at 1000 Hz, is only slightly attenuated, since f_1 is less than cutoff frequency f_g=2000 Hz. This

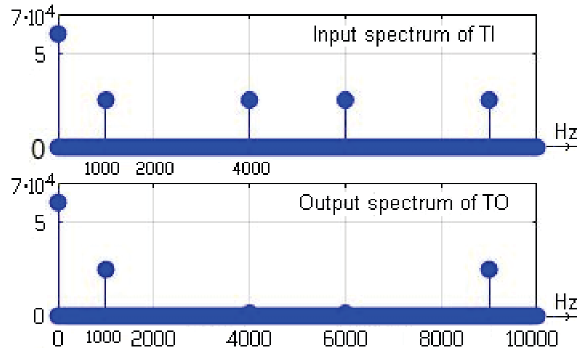

Fig. 3. *The input spectrum of TI and the output spectrum of TO.*

agrees with magnitude of the IIR FLL_3 frequency response shown in Fig. 2, since at f_1=1000 Hz, the attenuation is close to zero. At the same time signal S_2 at 4000 Hz is suppressed for about -24.5 dB in Fig. 2, because f_2=4000 Hz is greater than cutoff frequency f_g. It can be seen in Fig. 3, that the zero component at the frequency close to zero is not attenuated, what is also in agreement with the magnitude of IIR FLL_3, shown in Fig. 2. A more complete description regarding the zero component are presented in ref. [1] and [2].

CONCLUSION

Unlike refs. [1] and [2] which describe the new kind of FIR digital filters based on the processing of the input periods only, this article presents the design of a new kind of an IIR digital filter, based on the processing of the input and output periods. Both of them use the theory, respectively, of the classical FIR and IIR digital filters. They are both of them intended for the filtering of impulse signal periods.

This article represents an important contribution to the theory and application of new kind of IIR FLL digital filters, based on FLL. The shown adaption for the third order FLL to function as an IIR digital filter, using the theory of the classical IIR digital filters, can be applied to a FLL of any order.

This article opened the wide possibilities for the usage of IIR FLL digital filters in electronics, telecommunications, control and measurements, which use the different forms of periodic and non-periodic pulse signals. There is an obvious need to filter them in some of the applications.

However, the shown mathematical process of adaption can be very long and complex procedure, especially for a FLL of very high order, which is expected to be used in filtering. Because of that, in the next step, it is necessary to develop all the necessary equations, used in this adaptation, for FLL of the N-th order. This will enable short, simple and safe adaptation, which will be almost reduced to the development of a classical IIR digital filter.

Acknowledgements

This article was supported by the Ministry of Science and Technology of the Republic of Serbia within the project TR 32047.

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