OPTIMIZATION OF THE LANCZOS KERNEL PARAMETER FOR INTERPOLATION IMAGE

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Abstract
In this paper, the parameter optimization of the 1P Lanczos interpolation kernel is performed. The optimal value of the parameter was determined experimentally, by interpolating some test images. The accuracy of the Lanczos interpolation kernel was measured using MSE. For the purpose of comparative analysis interpolation was performed the test images with the implementation of the 1P Keys and Dodgson kernels. The results are presented graphically and tabularly.

Keywords: convolution, interpolation, parametric convolution interpolation kernel, optimal value parameter.

1. INTRODUCTION

The interpolation theory has been drawing attention since the early days. The word ’interpolation’ originates from Latin verb ’interpolare’ and means ’inserting new members between given members’. In English literature, the word ’interpolation’ is first encountered and used since 1612. and it’s used in sense of exchanging (text) after inserting a new one. Newton’s geometrical curve which pass through any given points descriptions, 1675 are the very basis of development of the interpolational methods.’ During the 19th Century, the interpolation methods based on polynomials had been intensively studied. Schoneberg 1946 defined basis functions which enable interpolation of equdistant data [1]. During 1970 interpolation techniques based on cubic convolution had been further developed [2], [3]. Researches about convolution interpolation are involved in many works [4] – [8]. Convolution interpolation is realised with the appliance of interpolation convolution kernel. Different interpolation kernels enable different accuracy and efficacy of interpolation algorithms. Velocity of interpolation algorithms execution and their numerical accuracy are directly linked with choice of interpolation kernel In order to increase the estimate of the interpolated value, the parameterization of the kernel was performed. The kernel parameter can be determined according to some criteria so as to obtain the highest accuracy of the estimate, ie to minimize the interpolation error. Thus, the determined value of the kernel parameter becomes optimal.

In this paper, the optimal value of one-parameter (1P) Lanczos kernel is decided by experimental ways, at interpolation of some test images . For each test image, the optimal value of kernel parameter, $\alpha_{opt.}$ is determined. For the purpose of comparative analysis, interpolation was performed in test images using the 1P Keys interpolation kernel [4] and the Dodgson kernel [5]. The accuracy of the interpolation kernel was measured using the mean square error (MSE) between the accurate and the interpolated value.

In the folowwing, 1P Lanczos interpolation kernel, which is based on the *sinc* function, is described. An experiment and an algorithm are described by which the optimal value of the kernel parameter is determined. The results are presented using tables and graphs.

2. LANCZOS INTERPOLATION KERNEL

1P Lanczos interpolation kernel is defined with[9]:

$$L(x)=\begin{cases}\dfrac{\sin(\pi x)}{\pi x}\cdot\dfrac{\sin((\pi x)/\alpha)}{(\pi x)/\alpha} & , \quad 0\le|x|<\alpha \\ 0 & , \quad |x|\ge\alpha\end{cases} \quad (1)$$

where is α kernel parameter.

In Fig. 1 1P Lanczos interpolation kernel is shown for some parameter values α.

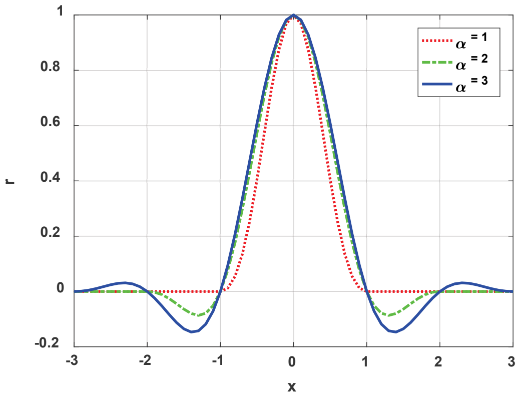


Fig. 1. *Lanczos interpolation kernel for different parameter value α.*

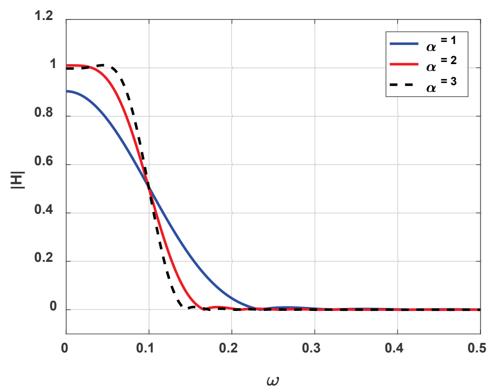


Fig. 2. *Spectral characteristics Lanczos kernel for different parameter value α.*

3. EXPERIMENTAL RESULTS AND ANALYSIS

The optimal parameter value α and the minimal mean square error of interpolation is determined by experimental ways at image interpolation case is determined.

3.1. The Experiment

Algorithm for determining optimal value of kernel parameter consists of following steps:

Input: Test image X dimension MxN, kernel r, length L, α_{min}, α_{max}, $\Delta\alpha$.

Output: $\alpha_{opt.}$

Step 1: Construction one-dimensional sequence $x_{M \cdot N}$ by connecting rows of the matrix X.

FOR $\alpha = \alpha_{min}: \Delta\alpha: \alpha_{max}$

Step 2: Construction of the kernel r in function α.

FOR $i = 1 : M \cdot N - (L+2)$

Step 3: Estimation $\hat{x}(i+L-1)$ by applying convolution with Lanczos kernel.

$$\hat{x}(i+L-1) = [x_i, x_{i+L-2}, x_{i+L}, x_{i+L+2}] \otimes r_\alpha$$

Step 4: Estimation the interpolation error:

$$e_{\alpha,i} = x(i+L\text{-}1) - \hat{x}(i+L-1)$$

END i

Step 5: Determination of the mean square error,

$$MSE_\alpha = \frac{1}{MN\text{-}(L+2)} \sum_{k=1}^{MN\text{-}(L+2)} \left| e_{\alpha,k} \right|^2$$

END α

Step 6: Determination optimal value, α_{opt} of kernel parameter:

$$\alpha_{opt} = \arg\min_{\alpha}(MSE_\alpha).$$

3.2 The Base

The base consists of $K = 8$ standard test images: *Lena, Pappers, Goldhill, Camerman, Boats, Barbara, Baboon, Cat.*

a) Lena

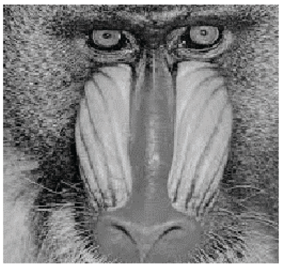
b) Peppers

c) Goldhill

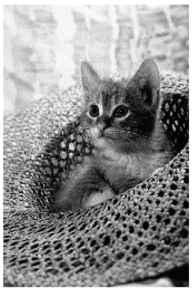
d) Camerman

e) Boats

f) Barbara



g) Baboon



h) Cat

Fig. 3. *The base of test images.*

3.3 Results

By Lanczos kernel application at interpolating some test images, the results were obtained for α_{opt} and MSE_{min} which are shown in Table 1 and in Fig. 4. For comparative analysis, the interpolation is done on test images also by using 1P Keys kernel and quadratic Dodgson kernel The results for α_{opt} and MSE_{min} are shown in Table 2 and Table 3. Mean value of all oprimal parameters and minimal values of MSE are shown in all tables.

Table 1*. Minimal mean square error for Lanczos kernel for test images from the base.*

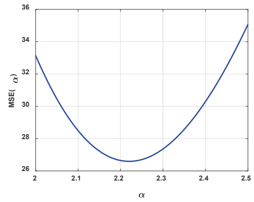
Image	α_{opt}	MSE_{min}
Lena	2.2200	26.5959
Peppers	2.2040	98.6184
Goldhill	1.3370	35.7681
Cameraman	1.3370	347.4825
Boats	1.3380	37.7240
Barbara	2.2050	31.7950
Baboon	1.3320	91.5108
Cat	1.3380	74.4397
	$\overline{\alpha}_{opt}$	$\overline{MSE}_{min}$
	1.6639	92.9918

Table 2*. Minimal mean square error for 1P Keys kernel for test images from the base.*

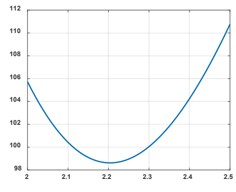
Image	α_{opt}	MSE_{min}
Lena	-0.7000	26.5724
Peppers	-0.4000	97.0203
Goldhill	0	35.7683
Cameraman	-0.1000	347.1408
Boats	-0.1000	37.6834
Barbara	-0.5000	31.6406
Baboon	0.9000	86.6896
Cat	-0.2000	73.9280
	$\overline{\alpha}_{opt}$	$\overline{MSE}_{min}$
	-0.1375	92.0554

Table 3*. Minimal mean square error for Dodgson for test images from the base .*

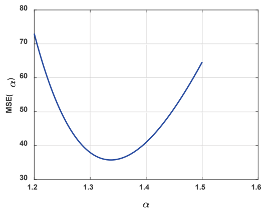
Image	α_{opt}	MSE_{min}
Lena	-2.7000	33.7984
Peppers	2.7000	100.1820
Goldhill	1.1000	35.7683
Cameraman	-1.6000	347.4828
Boats	-2.7000	37.7242
Barbara	1.1000	35.1087
Baboon	-1.7000	91.5440
Cat	0.8000	74.4423
	$\overline{\alpha}_{opt}$	$\overline{MSE}_{min}$
	-0.375	94.5036



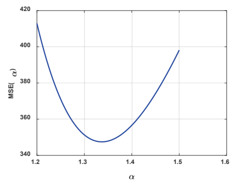
a) Lena



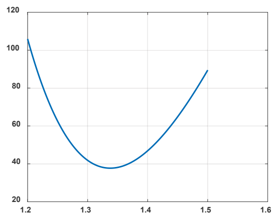
b)Peppers



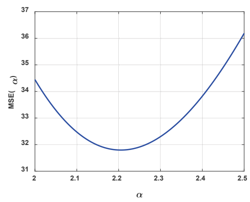
c) Goldhill



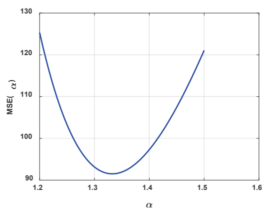
d) Cameraman



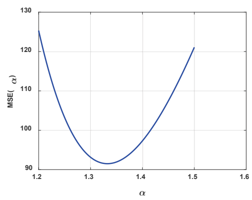
e) Boats



f) Barbara



g) Baboon



h) Cat

Fig.4. *Dependence of MSE kernel parameter at some test images.*

By conducting statistics analysis of parameter optimal values from Tbl. 1, mean value μ and variance σ^2 is determined. On the basis of μ i σ^2 , Gauss normal function, $p(\alpha)$ is determined and shown on the Fig. 5.

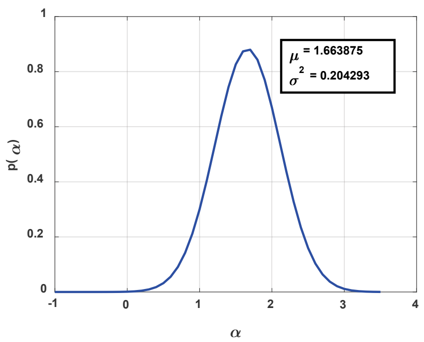


Fig5. *Function of Gauss normal distribution of optimal parameter values for test images.*

3.4 Comparative analysis

Based on the results shown on Fig. 2 and Table 1, Table 2 and Table 3, it is concluded that:

a) Range of optimal parameter values of the Lanczos kernel calculated for test images (Tbl. 1) $\alpha_{opt} \in [1.332 - 2.22]$ and mean value $\overline{\alpha_{opt}}$ = 1.6639.

b) Range of optimal parameter values of the Keys kernel calculated for test images (Tbl. 2) $\alpha_{opt} \in [-0.7 - 0]$ and mean value $\overline{\alpha_{opt}} = -0.1375$.

c) Range of optimal parameter values of the Dodgson kernel calculated for test images (Tbl. 3) $\alpha_{opt} \in [-2.7 - 1.1]$ and mean value $\overline{\alpha_{opt}} = -0.1375$

d) By the comparison of MSE, it can be concluded that the MSE obtained by intepolation using the 1P Lanczos kernel in relation to the MSE obtained by interpolation using the 1P Keys kernel, is $\overline{MSE}_{\min}\_L / \overline{MSE}_{\min}\_K = 92.9918 / 92.055 =$ 1.0102 times greater.

e) By the comparison of MSE, it can be concluded that the MSE obtained by intepolation using the 1P Lanczos kernel in relation to the MSE obtained by interpolation using the Dodgson kernel is $\overline{MSE}_{\min}\_L / \overline{MSE}_{\min}\_D = 92.9918 / 94.536 =$ 0.984 times lesser.

4. CONCLUSION

In this paper, the results of application 1P Lanczos kernel at image interpolation were shown. The optimal value of kernel parameter is defined by experimental ways α_{opt} = 1.6639. Kernel efficiency is calculated by MSE. By the analysis of MSE it can be concluded that the 1P Lanczos kernel is more efficient than Dodgson kernel, while it is less efficient than 1P Keys kernel.

REFERENCE

[1] I. J. Schoenberg, “Contributions to the problem of approximation of equidistant data by analytic functions. Part A—on the problem of smoothing or graduation. A first class of analytic approximation formulae,” *Quart. Appl. Math.*, vol. IV, no. 1, pp. 45–99, 1946.

[2] S. S. Rifman "Digital rectification of ERTS multispectral imagery", in *Proceedings of the Symposium on Significant Results Obtained from the Earth Resources Technology Satelite-1*, Vol. 1, section B, NASA SP-327, pp. 1131-1142, 1973.

[3] Hou H, Andrews H. 1978. Cubic splines for image interpolation and digital filtering. *IEEE Trans Acoust*. 26(6):508–517.

[4] R. G. Keys, "Cubic convolution interpolation for digital image processing", *IEEE Trans. Acout. Speech, & Signal Processing*, Vol. ASSP-29, pp. 1153-1160, Dec. 1981.

[5] A. Dodgson, "Quadratic interpolation for image resampling", *IEEE Trans. Image Processing*, Vol. 6, no.9, pp. 1322-1326, Sept. 1997.

[6] N. Savić, Z. Milivojević and D. Brodić, "Estimation of frequency of a signal by means of interpolation with a quadratic convolution kernel", *ETF Journal of Electrical Engineering*, Vol. 20, pp. 40-49, 2014.

[7] K. S. Park, R. A. Schowengerdt, "Image reconstruction by parametric cubic convolution," *Computer Vision, Graphics & Image Processing*, Vol. 23, pp. 258-272, 1983.

[8] E. Mejering. K. Zuiderveld, M. Viegever, "Image Reconstruction by Convolution with Symmetrical Piecewise nth-Order Polynomial Kernels", *IEEE Transactions on Image Processing*, Vol. 8, No. 2, pp. 192-201, Feb. 1999.

[9] P. Getreuer "Linear Methods for Image Interpolation", *Image Processing On Line*, vol. 1, pp. 238-259, 2011.